1. Which of the following is correct
(a) $\tan 1 > \tan 2$ (b) $\tan 1 = \tan 2$
(c) $\tan 1 < \tan 2$ (d) $\tan 1 = 1$

2. Which of the following relations is correct **[WB JEE 1991]**
(a) $\sin 1 < \sin 1^\circ$ (b) $\sin 1 > \sin 1^\circ$
(c) $\sin 1 = \sin 1^\circ$ (d) $\frac{\pi}{180}\sin\ 1 = \sin\ 1^o$

3. $\tan 1^\circ \tan 2^\circ \tan 3^\circ \tan 4^\circ .... \tan 89^\circ =$ **[MP PET 1998, 2001; AMU 1999; Pb. CET 1994]**
(a) 1 (b) 0
(c) ∞ (d) 1/2

4. If $\sin\theta + \text{cosec}\theta = 2,$ the value of $\sin^{10}\theta + \text{cosec}^{10}\theta$ is **[MP PET 2004]**
(a) 10 (b) 2^{10}
(c) 2^9 (d) 2

5. If $\sin\theta + \text{cosec}\theta = 2$, then $\sin^2\theta + \text{cosec}^2\theta =$ **[MP PET 1992; MNR 1990; UPSEAT 2002]**
(a) 1 (b) 4
(c) 2 (d) None of these

6. If $\sin\theta + \cos\theta = m$ and $\sec\theta + \text{cosec}\theta = n$, then $n(m+1)(m-1) =$ **[MP PET 1986]**
(a) m (b) n
(c) $2m$ (d) $2n$

7. If $\sin\theta + \cos\theta = 1$, then $\sin\theta\cos\theta =$ **[Karnataka CET 1998]**
(a) 0 (b) 1
(c) 2 (d) 1/2

8. If $\sin\theta = \frac{24}{25}$ and θ lies in the second quadrant, then $\sec\theta + \tan\theta =$ **[MP PET 1997]**
(a) – 3 (b) – 5
(c) – 7 (d) – 9

9. If $\text{cosec}\,A + \cot A = \frac{11}{2}$, then $\tan A =$ **[Roorkee 1995]**
(a) $\frac{21}{22}$ (b) $\frac{15}{16}$
(c) $\frac{44}{117}$ (d) $\frac{117}{43}$

10. If $5\tan\theta = 4,$ then $\frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 2\cos\theta} =$ **[Karnataka CET 1998]**
(a) 0 (b) 1
(c) 1/6 (d) 6

11. If $\tan\theta = \frac{20}{21}$, $\cos\theta$ will be **[MP PET 1994]**
(a) $\pm\frac{20}{41}$ (b) $\pm\frac{1}{21}$
(c) $\pm\frac{21}{29}$ (d) $\pm\frac{20}{21}$

12. If $\sin x = \frac{-24}{25}$, then the value of $\tan x$ is **[UPSEAT 2003]**
(a) $\frac{24}{25}$ (b) $\frac{-24}{7}$
(c) $\frac{25}{24}$ (d) None of these

13. If $\tan\theta = \frac{-4}{3}$, then $\sin\theta =$ **[IIT 1979; Pb. CET 1995; Orissa JEE 2002]**
(a) – 4/5 but not 4/5 (b) – 4/5 or 4/5
(c) 4/5 but not – 4/5 (d) None of these

14. If $\sin\theta = -\frac{1}{\sqrt{2}}$ and $\tan\theta = 1,$ then θ lies in which quadrant
(a) First (b) Second
(c) Third (d) Fourth

15. If $\sin\theta = \frac{-4}{5}$ and θ lies in the third quadrant, then $\cos\frac{\theta}{2} =$
(a) $\frac{1}{\sqrt{5}}$ (b) $-\frac{1}{\sqrt{5}}$
(c) $\sqrt{\frac{2}{5}}$ (d) $-\sqrt{\frac{2}{5}}$

16. If $\sin(\alpha - \beta) = \frac{1}{2}$ and $\cos(\alpha + \beta) = \frac{1}{2}$, where α and β are positive acute angles, then
(a) $\alpha = 45^\circ, \beta = 15^\circ$ (b) $\alpha = 15^\circ, \beta = 45^\circ$
(c) $\alpha = 60^\circ, \beta = 15^\circ$ (d) None of these

17. If $\tan\theta = -\frac{1}{\sqrt{10}}$ and θ lies in the fourth quadrant, then $\cos\theta =$
(a) $1/\sqrt{11}$ (b) $-1/\sqrt{11}$
(c) $\sqrt{\frac{10}{11}}$ (d) $-\sqrt{\frac{10}{11}}$

18. $(m+2)\sin\theta + (2m-1)\cos\theta = 2m+1,$ if
(a) $\tan\theta = \frac{3}{4}$ (b) $\tan\theta = \frac{4}{3}$
(c) $\tan\theta = \frac{2m}{m^2+1}$ (d) None of these

19. If A lies in the second quadrant and $3\tan A + 4 = 0,$ the value of $2\cot A - 5\cos A + \sin A$ is equal to **[Pb. CET 2000]**
(a) $\frac{-53}{10}$ (b) $\frac{-7}{10}$
(c) $\frac{7}{10}$ (d) $\frac{23}{10}$

20. If $\sin x + \sin y = 3(\cos y - \cos x)$, then the value of $\frac{\sin 3x}{\sin 3y}$ is

(a) 1 (b) – 1
(c) 0 (d) None of these

21. If $\sin A, \cos A$ and $\tan A$ are in G.P., then $\cos^3 A + \cos^2 A$ is equal to

(a) 1 (b) 2
(c) 4 (d) None of these

22. If θ lies in the second quadrant, then the value of $\sqrt{\left(\frac{1-\sin\theta}{1+\sin\theta}\right)} + \sqrt{\left(\frac{1+\sin\theta}{1-\sin\theta}\right)}$

(a) $2\sec\theta$ (b) $-2\sec\theta$
(c) $2\text{cosec}\theta$ (d) None of these

23. $\frac{\sin\theta}{1-\cot\theta} + \frac{\cos\theta}{1-\tan\theta} =$ **[Karnataka CET 1998]**

(a) 0 (b) 1
(c) $\cos\theta - \sin\theta$ (d) $\cos\theta + \sin\theta$

24. If $\tan\theta + \sec\theta = e^x$, then $\cos\theta$ equals **[AMU 2002]**

(a) $\frac{(e^x + e^{-x})}{2}$ (b) $\frac{2}{(e^x + e^{-x})}$
(c) $\frac{(e^x - e^{-x})}{2}$ (d) $\frac{(e^x - e^{-x})}{(e^x + e^{-x})}$

25. If $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$, then $\cos\theta + \sin\theta$ is equal to **WB JEE 1988]**

(a) $\sqrt{2}\cos\theta$ (b) $\sqrt{2}\sin\theta$
(c) $2\cos\theta$ (d) $-\sqrt{2}\cos\theta$

26. If $\sec\theta + \tan\theta = p$, then $\tan\theta$ is equal to **[MP PET 1994]**

(a) $\frac{2p}{p^2-1}$ (b) $\frac{p^2-1}{2p}$
(c) $\frac{p^2+1}{2p}$ (d) $\frac{2p}{p^2+1}$

27. If $x = \sec\theta + \tan\theta$, then $x + \frac{1}{x} =$ **[MP PET 1986]**

(a) 1 (b) $2\sec\theta$
(c) 2 (d) $2\tan\theta$

28. If $x + \frac{1}{x} = 2\cos\alpha$, then $x^n + \frac{1}{x^n} =$ **[Karnataka CET 2004]**

(a) $2^n\cos\alpha$ (b) $2^n\cos n\alpha$
(c) $2i\sin n\alpha$ (d) $2\cos n\alpha$

29. If $\cos\theta = \frac{1}{2}\left(x + \frac{1}{x}\right)$, then $\frac{1}{2}\left(x^2 + \frac{1}{x^2}\right) =$ **[AMU 1998]**

(a) $\sin 2\theta$ (b) $\cos 2\theta$
(c) $\tan 2\theta$ (d) $\sec 2\theta$

30. The value of $e^{\log_{10}\tan 1° + \log_{10}\tan 2° + \log_{10}\tan 3° + ...+\log_{10}\tan 89°}$ is

(a) 0 (b) e
(c) $1/e$ (d) None of these

31. $\cot x - \tan x =$ **[MP PET 1986]**

(a) $\cot 2x$ (b) $2\cot^2 x$
(c) $2\cot 2x$ (d) $\cot^2 2x$

32. $\frac{1+\sin A - \cos A}{1+\sin A + \cos A} =$

(a) $\sin\frac{A}{2}$ (b) $\cos\frac{A}{2}$
(c) $\tan\frac{A}{2}$ (d) $\cot\frac{A}{2}$

33. $\frac{2\sin\theta\tan\theta(1-\tan\theta) + 2\sin\theta\sec^2\theta}{(1+\tan\theta)^2} =$ **[Roorkee 1975]**

(a) $\frac{\sin\theta}{1+\tan\theta}$ (b) $\frac{2\sin\theta}{1+\tan\theta}$
(c) $\frac{2\sin\theta}{(1+\tan\theta)^2}$ (d) None of these

34. The value of the expression $1 - \frac{\sin^2 y}{1+\cos y} + \frac{1+\cos y}{\sin y} - \frac{\sin y}{1-\cos y}$ is equal to

(a) 0 (b) 1
(c) $\sin y$ (d) $\cos y$

35. If $2y\cos\theta = x\sin\theta$ and $2x\sec\theta - y\,\text{cosec}\,\theta = 3$, then $x^2 + 4y^2 =$ **[WB JEE 1988]**

(a) 4 (b) – 4
(c) ± 4 (d) None of these

36. If $\tan A + \cot A = 4$, then $\tan^4 A + \cot^4 A$ is equal to **[Kerala (Engg.) 2002]**

(a) 110 (b) 191
(c) 80 (d) 194

37. If $x = \sec\phi - \tan\phi, y = \text{cosec}\phi + \cot\phi$, then

(a) $x = \frac{y+1}{y-1}$ (b) $x = \frac{y-1}{y+1}$
(c) $y = \frac{1-x}{1+x}$ (d) None of these

38. If $\tan\theta = \frac{x\sin\phi}{1-x\cos\phi}$ and $\tan\phi = \frac{y\sin\theta}{1-y\cos\theta}$, then $\frac{x}{y} =$ **[MP PET 1991]**

(a) $\frac{\sin\phi}{\sin\theta}$ (b) $\frac{\sin\theta}{\sin\phi}$
(c) $\frac{\sin\phi}{1-\cos\theta}$ (d) $\frac{\sin\theta}{1-\cos\phi}$

39. If $p=\frac{2\sin\theta}{1+\cos\theta+\sin\theta}$, and $q=\frac{\cos\theta}{1+\sin\theta}$, then **[MP PET 2001]**

(a) $pq=1$ (b) $\frac{q}{p}=1$

(c) $q-p=1$ (d) $q+p=1$

40. If $\tan\theta+\sin\theta=m$ and $\tan\theta-\sin\theta=n$, then **[IIT 1970]**

(a) $m^2-n^2=4mn$ (b) $m^2+n^2=4mn$

(c) $m^2-n^2=m^2+n^2$ (d) $m^2-n^2=4\sqrt{mn}$

41. If $\tan\theta=\frac{a}{b}$, then $\frac{\sin\theta}{\cos^8\theta}+\frac{\cos\theta}{\sin^8\theta}=$ **[WB JEE 1986]**

(a) $\pm\frac{(a^2+b^2)^4}{\sqrt{a^2+b^2}}\left(\frac{a}{b^8}+\frac{b}{a^8}\right)$

(b) $\pm\frac{(a^2+b^2)^4}{\sqrt{a^2+b^2}}\left(\frac{a}{b^8}-\frac{b}{a^8}\right)$

(c) $\pm\frac{(a^2-b^2)^4}{\sqrt{a^2+b^2}}\left(\frac{a}{b^8}+\frac{b}{a^8}\right)$

(d) $\pm\frac{(a^2-b^2)^4}{\sqrt{a^2-b^2}}\left(\frac{a}{b^8}-\frac{b}{a^8}\right)$

42. If $a\cos\theta+b\sin\theta=m$ and $a\sin\theta-b\cos\theta=n$, then $a^2+b^2=$

(a) $m+n$ (b) m^2-n^2

(c) m^2+n^2 (d) None of these

43. If $x=a\cos^3\theta, y=b\sin^3\theta$, then

(a) $\left(\frac{a}{x}\right)^{2/3}+\left(\frac{b}{y}\right)^{2/3}=1$

(b) $\left(\frac{b}{x}\right)^{2/3}+\left(\frac{a}{y}\right)^{2/3}=1$

(c) $\left(\frac{x}{a}\right)^{2/3}+\left(\frac{y}{b}\right)^{2/3}=1$

(d) $\left(\frac{x}{b}\right)^{2/3}+\left(\frac{y}{a}\right)^{2/3}=1$

44. If $\cot\theta+\tan\theta=m$ and $\sec\theta-\cos\theta=n$, then which of the following is correct

(a) $m(mn^2)^{1/3}-n(nm^2)^{1/3}=1$

(b) $m(m^2n)^{1/3}-n(mn^2)^{1/3}=1$

(c) $n(mn^2)^{1/3}-m(nm^2)^{1/3}=1$

(d) $n(m^2n)^{1/3}-m(mn^2)^{1/3}=1$

45. $\sin^6\theta+\cos^6\theta+3\sin^2\theta\cos^2\theta=$ **[MP PET 1995, 2002; DCE 2005]**

(a) 0 (b) –1

(c) 1 (d) None of these

46. The value of $2(\sin^6\theta+\cos^6\theta)-3(\sin^4\theta+\cos^4\theta)+1$ is **[MP PET 1997; UPSEAT 2002]**

(a) 2 (b) 0

(c) 4 (d) 6

47. If $\sin x+\sin^2 x=1$, then the value of $\cos^{12}x+3\cos^{10}x+3\cos^8 x+\cos^6 x-2$ is equal to **[Pb. CET 2002]**

(a) 0 (b) 1

(c) – 1 (d) 2

48. If $\cos x+\cos^2 x=1$, then the value of $\sin^2 x+\sin^4 x$ is

(a) 1 (b) – 1

(c) 0 (d) 2

49. If $\sin x+\sin^2 x=1$, then $\cos^8 x+2\cos^6 x+\cos^4 x=$

(a) 0 (b) – 1

(c) 2 (d) 1

50. If $x\sin^3\alpha+y\cos^3\alpha=\sin\alpha\cos\alpha$ and $x\sin\alpha-y\cos\alpha=0$, then $x^2+y^2=$ **[WB JEE 1984]**

(a) – 1 (b) ±1

(c) 1 (d) None of these

51. If $(1+\sin A)(1+\sin B)(1+\sin C)$ $=(1-\sin A)(1-\sin B)(1-\sin C)$, then each side is equal to

(a) $\pm\sin A\sin B\sin C$ (b) $\pm\cos A\cos B\cos C$

(c) $\pm\sin A\cos B\cos C$ (d) $\pm\cos A\sin B\sin C$

52. If $(\sec\alpha+\tan\alpha)(\sec\beta+\tan\beta)(\sec\gamma+\tan\gamma)$ $=\tan\alpha\tan\beta\tan\gamma$, then $(\sec\alpha-\tan\alpha)(\sec\beta-\tan\beta)$ $(\sec\gamma-\tan\gamma)=$ **[Kurukshetra CEE 1998]**

(a) $\cot\alpha\cot\beta\cot\gamma$ (b) $\tan\alpha\tan\beta\tan\gamma$

(c) $\cot\alpha+\cot\beta+\cot\gamma$ (d) $\tan\alpha+\tan\beta+\tan\gamma$

53. If $\sin\theta_1+\sin\theta_2+\sin\theta_3=3$, then $\cos\theta_1+\cos\theta_2+\cos\theta_3=$ **[EAMCET 1994]**

(a) 3 (b) 2

(c) 1 (d) 0

54. If $\sin^2\theta=\frac{x^2+y^2+1}{2x}$, then *x* must be **[UPSEAT 2004]**

(a) – 3 (b) – 2

(c) 1 (d) None of these

55. If $\tan\theta-\cot\theta=a$ and $\sin\theta+\cos\theta=b$, then $(b^2-1)^2(a^2+4)$ is equal to **[WB JEE 1979]**

(a) 2 (b) – 4

(c) ± 4 (d) 4

56. If $\tan^2\alpha\tan^2\beta+\tan^2\beta\tan^2\gamma+\tan^2\gamma\tan^2\alpha$ $+2\tan^2\alpha\tan^2\beta\tan^2\gamma=1$, then the value of $\sin^2\alpha+\sin^2\beta+\sin^2\gamma$ is

(a) 0 (b) – 1

(c) 1 (d) None of these

57. $\cos 1^\circ.\cos 2^\circ.\cos 3^\circ.........\cos 179^\circ =$ **[Karnataka CET 1999; DCE 2005]**

(a) 0 (b) 1

(c) 2 (d) $\frac{1}{2}$

58. The value of $\frac{\cot 54^\circ}{\tan 36^\circ}+\frac{\tan 20^\circ}{\cot 70^\circ}$ is **[Karnataka CET 1999]**

(a) 2 (b) 3

(c) 1 (d) 0

59. The value of $\sin 10^\circ+\sin 20^\circ+\sin 30^\circ+...+\sin 360^\circ$ is **[Pb. CET 2003]**

(a) 1 (b) 0

(c) – 1 (d) None of these

60. $\cos 1^\circ+\cos 2^\circ+\cos 3^\circ+.....+\cos 180^\circ =$ **[Karnataka CET 2003]**

(a) 0 (b) 1

(c) – 1 (d) 2

61. If $\alpha = 22^\circ 30'$, then $(1+\cos\alpha)(1+\cos 3\alpha)(1+\cos 5\alpha)(1+\cos 7\alpha)$ equals **[AMU 1999]**

(a) 1/8 (b) 1/4

(c) $\frac{1+\sqrt{2}}{2\sqrt{2}}$ (d) $\frac{\sqrt{2}-1}{\sqrt{2}+1}$

62. The value of $6(\sin^6\theta+\cos^6\theta)-9(\sin^4\theta+\cos^4\theta)+4$ is **[MP PET 2001]**

(a) –3 (b) 0

(c) 1 (d) 3

63. $\sin 15^\circ+\cos 105^\circ =$ **[MP PET 1992]**

(a) 0 (b) $2\sin 15^\circ$

(c) $\cos 15^\circ+\sin 15^\circ$ (d) $\sin 15^\circ-\cos 15^\circ$

64. The value $\cos 105^\circ+\sin 105^\circ$ is **[MNR 1975]**

(a) $\frac{1}{2}$ (b) 1

(c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$

65. The value of $\cos y\cos\left(\frac{\pi}{2}-x\right)-\cos\left(\frac{\pi}{2}-y\right)\cos x+\sin y\cos\left(\frac{\pi}{2}-x\right)+\cos x\sin\left(\frac{\pi}{2}-y\right)$ is zero, if

(a) $x=0$ (b) $y=0$

(c) $x=y$ (d) $x=n\pi-\frac{\pi}{4}+y,\ (n\in I)$

66. $\sin\left(\frac{\pi}{10}\right)\sin\left(\frac{3\pi}{10}\right)=$ **[MNR 1984]**

(a) 1/2 (b) – 1/2

(c) 1/4 (d) 1

67. If $x\sin 45^\circ\cos^2 60^\circ=\frac{\tan^2 60^\circ \text{cosec} 30^\circ}{\sec 45^\circ\cot^2 30^\circ}$, then $x=$

(a) 2 (b) 4

(c) 8 (d) 16

68. If $A=130^\circ$ and $x=\sin A+\cos A$, then **[CET 1989]**

(a) $x>0$ (b) $x<0$

(c) $x=0$ (d) $x\le 0$

69. $\cos A+\sin(270^\circ+A)-\sin(270^\circ-A)+\cos(180^\circ+A)=$ **[MP PET 1990]**

(a) – 1 (b) 0

(c) 1 (d) None of these

70. If $\pi<\alpha<\frac{3\pi}{2}$, then $\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}+\sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}}=$

(a) $\frac{2}{\sin\alpha}$ (b) $-\frac{2}{\sin\alpha}$

(c) $\frac{1}{\sin\alpha}$ (d) $-\frac{1}{\sin\alpha}$

71. $\tan\left(\frac{\pi}{4}+\theta\right)-\tan\left(\frac{\pi}{4}-\theta\right)=$

(a) $2\tan 2\theta$ (b) $2\cot 2\theta$

(c) $\tan 2\theta$ (d) $\cot 2\theta$

72. $\sin(\pi+\theta)\sin(\pi-\theta)\ \text{cosec}^2\theta=$ **[EAMCET 1980]**

(a) 1 (b) –1

(c) $\sin\theta$ (d) $-\sin\theta$

73. $\cot(45^\circ+\theta)\cot(45^\circ-\theta)=$ **[MNR 1973]**

(a) – 1 (b) 0

(c) 1 (d) ∞

74. $\tan A+\cot(180^\circ+A)+\cot(90^\circ+A)+\cot(360^\circ-A)$ **[MP PET 1992]**

(a) 0 (b) $2\tan A$

(c) $2\cot A$ (d) $2(\tan A-\cot A)$

75. $\tan\theta\sin\left(\frac{\pi}{2}+\theta\right)\cos\left(\frac{\pi}{2}-\theta\right)=$ **[EAMCET 1981]**

(a) 1 (b) 0

(c) $\frac{1}{\sqrt{2}}$ (d) None of these

76. If angle θ be divided into two parts such that the tangent of one part is k times the tangent of the other and ϕ is their difference, then $\sin\theta=$

(a) $\frac{k+1}{k-1}\sin\phi$ (b) $\frac{k-1}{k+1}\sin\phi$

(c) $\frac{2k-1}{2k+1}\sin\phi$ (d) None of these

77. If $x=y\cos\frac{2\pi}{3}=z\cos\frac{4\pi}{3}$, then $xy+yz+zx=$ **[EAMCET 1994]**

(a) – 1 (b) 0

(c) 1 (d) 2

78. Given that $\pi < \alpha < \frac{3\pi}{2}$, then the expression $\sqrt{(4\sin^4\alpha + \sin^2 2\alpha)} + 4\cos^2\left(\frac{\pi}{4} - \frac{\alpha}{2}\right)$ is equal to

(a) 2 (b) $2 + 4\sin\alpha$
(c) $2 - 4\sin\alpha$ (d) None of these

79. $\sin^2\frac{\pi}{8} + \sin^2\frac{3\pi}{8} + \sin^2\frac{5\pi}{8} + \sin^2\frac{7\pi}{8} =$

(a) 1 (b) – 1
(c) 0 (d) 2

80. $(\sec A + \tan A - 1)(\sec A - \tan A + 1) - 2\tan A =$ **[Roorkee 1972]**

(a) $\sec A$ (b) $2\sec A$
(c) 0 (d) 1

81. The value of $\tan(-945°)$ is **[MP PET 1997]**

(a) – 1 (b) – 2
(c) – 3 (d) – 4

82. If $\tan A = \frac{1}{2}, \tan B = \frac{1}{3}$, then $\cos 2A =$ **[CET 1989]**

(a) $\sin B$ (b) $\sin 2B$
(c) $\sin 3B$ (d) None of these

83. The value of $\cos A - \sin A$ when $A = \frac{5\pi}{4}$, is **[MP PET 1990]**

(a) $\sqrt{2}$ (b) $\frac{1}{\sqrt{2}}$
(c) 0 (d) 1

84. The value of $\cos(270° + \theta)\cos(90° - \theta) - \sin(270° - \theta)\cos\theta$ is **[Karnataka CET 2005]**

(a) 0 (b) −1
(c) 1/2 (d) 1

85. If $\cos(\alpha - \beta) = 1$ and $\cos(\alpha + \beta) = \frac{1}{e}$, $-\pi < \alpha, \beta < \pi$, then total number of ordered pair of (α, β) is **[IIT Screening 2005]**

(a) 0 (b) 1
(c) 2 (d) 4

Solutions

1. (a) It is obvious.

2. (b) The true relation is $\sin 1 > \sin 1°$

Since value of $\sin\theta$ is increasing $\left[0 \to \frac{\pi}{2}\right]$.

3. (a) $\tan 1° \tan 2° .... \tan 89°$
$= (\tan 1° \tan 89°)(\tan 2° \tan 88°)......$
$= 1 \times 1 \times 1.... = 1.$

4. (d) We have,
$\sin\theta + \text{cosec}\theta = 2 \Rightarrow \sin^2\theta + 1 = 2\sin\theta$
$\Rightarrow \sin^2\theta - 2\sin\theta + 1 = 0$
$\Rightarrow (\sin\theta - 1)^2 = 0 \Rightarrow \sin\theta = 1$
Required value of
$\sin^{10}\theta + \text{cosec}^{10}\theta = (1)^{10} + \frac{1}{(1)^{10}} = 2$.

5. (c)

$\sin^2\theta + \text{cosec}^2\theta = (\sin\theta + \text{cosec}\theta)^2 - 2\sin\theta\,\text{cosec}\theta$

$= (2)^2 - 2 = 4 - 2 = 2$, since $(\sin\theta + \text{cosec}\theta) = 2$.

6. (c) $n(m^2 - 1) = (\sec\theta + \text{cosec}\theta).2\sin\theta\cos\theta$
($\because m^2 = 1 + 2\sin\theta\cos\theta$)
$= \frac{\sin\theta + \cos\theta}{\sin\theta.\cos\theta}.2\sin\theta\cos\theta = 2m$.

7. (a) $\sin\theta + \cos\theta = 1$
Squaring on both sides, we get
$\Rightarrow \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 1$
$\therefore \sin\theta\cos\theta = 0$.

8. (c) $\sin\theta = \frac{24}{25} \Rightarrow \cos\theta = \frac{-7}{25}, \tan\theta = \frac{-24}{7}$

$\therefore \sec\theta + \tan\theta = \frac{-25}{7} + \frac{-24}{7} = -7$

9. (c) $\text{cosec}\,A + \cot A = \frac{11}{2} \Rightarrow \text{cosec}\,A - \cot A = \frac{2}{11}$

Therefore $2\cot A = \frac{117}{22} \Rightarrow \tan A = \frac{44}{117}$.

10. (c) $5\tan\theta = 4 \Rightarrow \tan\theta = \frac{4}{5}$

$\therefore \sin\theta = \frac{4}{\sqrt{41}}$ and $\cos\theta = \frac{5}{\sqrt{41}}$

$$\frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 2\cos\theta} = \frac{5 \times \frac{4}{\sqrt{41}} - 3 \times \frac{5}{\sqrt{41}}}{5 \times \frac{4}{\sqrt{41}} + 2 \times \frac{5}{\sqrt{41}}}$$

$\frac{20-15}{20+10} = \frac{5}{30} = \frac{1}{6}$.

11. (c) $\tan\theta = \frac{20}{21} \Rightarrow \cos\theta = \pm\frac{21}{29}$.

12. (b) $\cos x = \sqrt{1 - \sin^2 x} = \sqrt{1 - \left(\frac{-24}{25}\right)^2} = \frac{7}{25}$

$\Rightarrow \tan x = \frac{\sin x}{\cos x} = \frac{-24}{7}$.

13. (b) Since $\text{cosec}^2\theta = 1 + \cot^2\theta = 1 + \frac{9}{16} = \frac{25}{16}$

$\left(\because \tan\theta = -\frac{4}{3}\right)$

$\sin^2\theta = \frac{1}{\text{cosec}^2\theta} = \frac{16}{25} \Rightarrow \sin\theta = \pm\frac{4}{5}$,

Both the values are acceptable, since $\tan\theta = -\frac{4}{3}$

i.e., θ lies in 2^{nd} or 4^{th} quadrant.

14. (c) $\sin\theta = -\frac{1}{\sqrt{2}}$ and $\tan\theta = 1$
$\Rightarrow \sin\theta = \sin 225° \Rightarrow \theta = 225°$
Since $\sin\theta$ is $-ve$ and $\tan\theta$ is $+ve$ in third quadrant.

15. (b) Given that $\sin\theta = -\frac{4}{5}$ and θ lies in the III quadrant.

$\Rightarrow \cos\theta = \sqrt{1 - \frac{16}{25}} = \pm\frac{3}{5}$

$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}} = \sqrt{\frac{1-3/5}{2}} = \pm\sqrt{\frac{1}{5}}$

But $\cos\frac{\theta}{2} = -\frac{1}{\sqrt{5}}$. since $\frac{\theta}{2}$ will be in II quadrant.

Hence $\cos\frac{\theta}{2} = -\frac{1}{\sqrt{5}}$.

16. (a) $\sin(\alpha - \beta) = \frac{1}{2} = \sin 30° \Rightarrow \alpha - \beta = 30°$ .....(i)

and $\cos(\alpha + \beta) = \frac{1}{2} \Rightarrow \alpha + \beta = 60°$ .....(ii)

Solving (i) and (ii), we get $\alpha = 45°$ and $\beta = 15°$.

Trick : In such type of problems, students should satisfy the given conditions with the values given in the options. Here $\alpha = 45°$ and $\beta = 15°$ satisfy both the conditions.

17. (c) We have $\tan\theta = -\frac{1}{\sqrt{10}}$, therefore θ is in IV quadrant. So $\cos\theta = +ve$.

Now $1 + \tan^2\theta = \sec^2\theta \Rightarrow 1 + \frac{1}{10} = \sec^2\theta$

$\Rightarrow \sec^2\theta = \frac{11}{10} \Rightarrow \cos\theta = \sqrt{\left(\frac{10}{11}\right)}$.

18. (b) Squaring the given relation and putting $\tan\theta = t$,

$$(m+2)^2 t^2 + 2(m+2)(2m-1)t + (2m-1)^2 = (2m+1)^2(1+t^2)$$

$$\Rightarrow 3(1-m^2)t^2 + (4m^2+6m-4)t - 8m = 0$$

$$\Rightarrow (3t-4)[(1-m^2)t + 2m] = 0,$$

which is true if $t = \tan\theta = \frac{4}{3}$ or $\tan\theta = \frac{2m}{m^2-1}$.

19. (d) $3\tan A + 4 = 0 \Rightarrow \tan A = -\frac{4}{3}$

$$\Rightarrow \sin A = \pm\frac{\tan A}{\sqrt{1+\tan^2 A}} = \pm\frac{-4/3}{\sqrt{1+16/9}} = \frac{4}{5}$$

($\because$ A is in 2nd quadrant)

and $\cos A = -\frac{3}{5}$. Thus, $2\cot A - 5\cos A + \sin A$

$$= 2\left(-\frac{3}{4}\right) - 5\left(-\frac{3}{5}\right) + \frac{4}{5} = \frac{23}{10}.$$

20. (b) We have $\sin x + \sin y = 3(\cos y - \cos x)$

$\Rightarrow \sin x + 3\cos x = 3\cos y - \sin y$ .....(i)

$\Rightarrow r\cos(x-\alpha) = r\cos(y+\alpha)$,

where $r = \sqrt{10}, \tan\alpha = \frac{1}{3}$

$\Rightarrow x - \alpha = \pm(y+\alpha) \Rightarrow x = -y$ or $x + y = 2\alpha$

Clearly, $x = -y$ satisfies (i);

$\therefore \frac{\sin 3x}{\sin 3y} = \frac{-\sin 3y}{\sin 3y} = -1$.

21. (a) We have $\sin A, \cos A$ and $\tan A$ are in $G.P.$

$$\cos^2 A = \sin A \tan A = \frac{\sin^2 A}{\cos A} \Rightarrow \cos^3 A - \sin^2 A = 0$$

Hence $\cos^3 A + \cos^2 A = \sin^2 A + \cos^2 A = 1$

22. (b) $\sqrt{\left(\frac{1-\sin\theta}{1+\sin\theta}\right)} + \sqrt{\left(\frac{1+\sin\theta}{1-\sin\theta}\right)}$ is the sum of two positive quantities and hence the result must be positive. But for $\frac{\pi}{2} < \theta < \pi$, we have the sum equal to $\frac{1-\sin\theta+1+\sin\theta}{\sqrt{1-\sin^2\theta}} = \frac{2}{\cos\theta}$; which is negative.

($\because$ $\cos\theta$ is negative for θ lying in 2nd quadrant). So the required positive value

$$= \frac{-2}{\cos\theta} = -2\sec\theta, \left(\frac{\pi}{2} < \theta < \pi\right).$$

23. (d) $\frac{\sin\theta}{1-\cot\theta} + \frac{\cos\theta}{1-\tan\theta}$

$$= \frac{\sin\theta.\sin\theta}{\sin\theta-\cos\theta} + \frac{\cos\theta.\cos\theta}{\cos\theta-\sin\theta}$$

$$= \frac{(\cos^2\theta-\sin^2\theta)}{(\cos\theta-\sin\theta)} = \cos\theta + \sin\theta.$$

24. (b) $\tan\theta + \sec\theta = e^x$ .....(i)

$\therefore$ $\sec\theta - \tan\theta = e^{-x}$ .....(ii)

From (i) and (ii),

$$2\sec\theta = e^x + e^{-x} \Rightarrow \cos\theta = \frac{2}{e^x+e^{-x}}.$$

25. (a) We have $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$

$$\Rightarrow \cos\theta = (\sqrt{2}+1)\sin\theta \Rightarrow (\sqrt{2}-1)\cos\theta = \sin\theta$$

$$\Rightarrow \sqrt{2}\cos\theta - \cos\theta = \sin\theta \Rightarrow \sin\theta + \cos\theta = \sqrt{2}\cos\theta.$$

26. (b) $\sec\theta + \tan\theta = p \Rightarrow \sec\theta - \tan\theta = \frac{1}{p}$

Subtracting second from first, we get

$$2\tan\theta = p - \frac{1}{p}$$

$$\Rightarrow \tan\theta = \frac{p^2-1}{2p}.$$

27. (b) Given that $x = \sec\theta + \tan\theta$

$$\Rightarrow x + \frac{1}{x} = \sec\theta + \tan\theta + \frac{1}{\sec\theta+\tan\theta}$$

$$= \sec\theta + \tan\theta + \sec\theta - \tan\theta = 2\sec\theta$$

28. (d) We have, $x + \frac{1}{x} = 2\cos\alpha$

$$x^2 + \frac{1}{x^2} + 2 = 4\cos^2\alpha.$$

$$x^2 + \frac{1}{x^2} = 4\cos^2\alpha - 2,$$

$$x^2 + \frac{1}{x^2} = 2(2\cos^2\alpha - 1) = 2\cos 2\alpha$$

Similarly $x^n + \frac{1}{x^n} = 2\cos n\alpha$.

29. (b) Given that $\cos\theta = \frac{1}{2}\left(x + \frac{1}{x}\right) \Rightarrow x + \frac{1}{x} = 2\cos\theta$

We know that $x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2$

$$= (2\cos\theta)^2 - 2 = 4\cos^2\theta - 2 = 2\cos 2\theta$$

$$\therefore \frac{1}{2}\left(x^2 + \frac{1}{x^2}\right) = \frac{1}{2} \times 2\cos 2\theta = \cos 2\theta$$

30. (d) We have

$$e^{\log_{10}\tan 1^o + \log_{10}\tan 2^o + \log_{10}\tan 3^o + ... + \log_{10}\tan 89^o}$$

$$= e^{\log_{10}(\tan 1^o \tan 2^o \tan 3^o ..... \tan 89^o)} = e^{\log_{10} 1} = e^o = 1$$

31. (c) $\cot x - \tan x = \frac{\cos x}{\sin x} - \frac{\sin x}{\cos x} = \frac{\cos^2 x - \sin^2 x}{\sin x \cos x}$

$$= \frac{2\cos 2x}{\sin 2x} = 2\cot 2x.$$

32. (c) $\dfrac{1+\sin A-\cos A}{1+\sin A+\cos A}$

$$=\frac{2\sin^2\frac{A}{2}+2\sin\frac{A}{2}\cos\frac{A}{2}}{2\cos^2\frac{A}{2}+2\sin\frac{A}{2}\cos\frac{A}{2}}$$

$$=\frac{2\sin\frac{A}{2}\left(\sin\frac{A}{2}+\cos\frac{A}{2}\right)}{2\cos\frac{A}{2}\left(\cos\frac{A}{2}+\sin\frac{A}{2}\right)}=\tan\frac{A}{2}.$$

Trick : Put $A=60^o$.

Then $\dfrac{1+(\sqrt{3}/2)-(1/2)}{1+(\sqrt{3}/2)+(1/2)}=\dfrac{1+\sqrt{3}}{3+\sqrt{3}}=\dfrac{1}{\sqrt{3}}$

which is given by option (c), *i.e.* $\tan\dfrac{60^o}{2}=\dfrac{1}{\sqrt{3}}$

Note : Students should remember at the time of assuming the values of *A, B, θ,* ..... *etc.* that, for the assumed values, the options must have different values.

33. (b) Given expression

$$=\frac{2\sin\theta}{(1+\tan\theta)^2}\{\tan\theta(1-\tan\theta)+\sec^2\theta\}$$

$$=\frac{2\sin\theta}{(1+\tan\theta)^2}\{\tan\theta-\tan^2\theta+1+\tan^2\theta\}$$

$$=\frac{2\sin\theta}{1+\tan\theta}.$$

34. (d) The expression can be written as

$$\frac{1+\cos y-\sin^2 y}{1+\cos y}+\frac{(1-\cos^2 y)-\sin^2 y}{\sin y(1-\cos y)}$$

$$=\frac{\cos y(1+\cos y)}{1+\cos y}+0=\cos y.$$

35. (a) Given that $2y\cos\theta=x\sin\theta$ .....(i)

and $2x\sec\theta-y\,\text{cosec}\,\theta=3$ .....(ii)

$$\Rightarrow \frac{2x}{\cos\theta}-\frac{y}{\sin\theta}=3$$

$$\Rightarrow 2x\sin\theta-y\cos\theta-3\sin\theta\cos\theta=0$$ .....(iii)

Solving (i) and (iii), we get $y=\sin\theta$ and $x=2\cos\theta$

Now, $x^2+4y^2=4\cos^2\theta+4\sin^2\theta$

$$=4(\cos^2\theta+\sin^2\theta)=4.$$

36. (d) $\tan A+\cot A=4$

$$\Rightarrow \tan^2 A+\cot^2 A+2\tan A\cot A=16$$

$$\Rightarrow \tan^2 A+\cot^2 A=14 \Rightarrow \tan^4 A+\cot^4 A+2=196$$

$$\Rightarrow \tan^4 A+\cot^4 A=194.$$

37. (b) We have $xy=(\sec\phi-\tan\phi)(\text{cosec}\,\phi+\cot\phi)$

$$=\frac{1-\sin\phi}{\cos\phi}\cdot\frac{1+\cos\phi}{\sin\phi}$$

$$\Rightarrow xy+1=\frac{1-\sin\phi+\cos\phi-\sin\phi\cos\phi+\sin\phi\cos\phi}{\cos\phi\sin\phi}$$

$$=\frac{1-\sin\phi+\cos\phi}{\cos\phi\sin\phi}$$ .....(i)

$$x-y=(\sec\phi-\tan\phi)-(\text{cosec}\,\phi+\cot\phi)$$

$$=\frac{1-\sin\phi}{\cos\phi}-\frac{1+\cos\phi}{\sin\phi}=\frac{\sin\phi-\sin^2\phi-\cos\phi-\cos^2\phi}{\cos\phi\sin\phi}$$

$$=\frac{\sin\phi-\cos\phi-1}{\cos\phi\sin\phi}$$ .....(ii)

Adding (i) and (ii) we get, $xy+1+(x-y)=0$

$$\Rightarrow x=\frac{y-1}{y+1}.$$

38. (b) We have $\tan\theta=\dfrac{x\sin\phi}{1-x\cos\phi}$

$$\Rightarrow \frac{1}{x}\tan\theta-\tan\theta\cos\phi=\sin\phi$$

$$\Rightarrow \frac{1}{x}=\frac{\sin\phi+\cos\phi\tan\theta}{\tan\theta} \text{ and } \tan\phi=\frac{y\sin\theta}{1-y\cos\theta}$$

$$\Rightarrow \tan\phi=\frac{\sin\theta}{\frac{1}{y}-\cos\theta}$$

$$\Rightarrow \frac{1}{y}\tan\phi-\tan\phi\cos\theta=\sin\theta$$

$$\Rightarrow \frac{1}{y}\tan\phi=\sin\theta+\tan\phi\cos\theta$$

$$\therefore \frac{1}{y}=\frac{\sin\theta+\tan\phi\cos\theta}{\tan\phi}$$

Now

$$\frac{x}{y}=\left[\frac{\tan\theta}{\sin\phi+\cos\phi\tan\theta}\right]\times\left[\frac{\sin\theta+\tan\phi\cos\theta}{\tan\phi}\right]$$

$$=\frac{\tan\theta}{\tan\phi}\left[\frac{\sin\theta+\cos\theta\frac{\sin\phi}{\cos\phi}}{\sin\phi+\cos\phi\frac{\sin\theta}{\cos\theta}}\right]=\frac{\tan\theta\cos\theta}{\tan\phi\cos\phi}=\frac{\sin\theta}{\sin\phi}$$

Aliter : $x\sin\phi=\tan\theta-x\cos\phi\tan\theta$

$$\Rightarrow x=\frac{\tan\theta}{\sin\phi+\cos\phi\tan\theta}$$

$$=\frac{\sin\theta}{\cos\theta\sin\phi+\cos\phi\sin\theta}=\frac{\sin\theta}{\sin(\theta+\phi)}$$

Similarly, $y=\dfrac{\sin\phi}{\sin(\theta+\phi)}$; $\therefore \dfrac{x}{y}=\dfrac{\sin\theta}{\sin\phi}$.

39. (d) $p = \frac{2\sin\theta}{1+\cos\theta+\sin\theta}$, $q = \frac{\cos\theta}{1+\sin\theta}$

$\Rightarrow p+q = \frac{\cos\theta}{1+\sin\theta} + \frac{2\sin\theta}{1+\sin\theta+\cos\theta} \Rightarrow p+q=1.$

40. (d) $(m+n) = 2\tan\theta,\ m-n = 2\sin\theta$

$\therefore\ m^2 - n^2 = 4\tan\theta.\sin\theta$ .....(i)

$4\sqrt{mn} = 4\sqrt{\tan^2\theta - \sin^2\theta} = 4\sin\theta.\tan\theta$ .....(ii)

From (i) and (ii), $m^2 - n^2 = 4\sqrt{mn}$.

41. (a) Given that $\tan\theta = \frac{a}{b}$

and $\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta} = \frac{b^2-a^2}{b^2+a^2}$

$\sin\theta = \pm\frac{a}{\sqrt{a^2+b^2}}$, $\cos\theta = \pm\frac{b}{\sqrt{a^2+b^2}}$

Now,

$$\frac{\sin\theta}{\cos^8\theta} + \frac{\cos\theta}{\sin^8\theta} = \frac{\left(\frac{a}{\sqrt{a^2+b^2}}\right)}{\left(\frac{b}{\sqrt{a^2+b^2}}\right)^8} + \frac{\left(\frac{b}{\sqrt{a^2+b^2}}\right)}{\left(\frac{a}{\sqrt{a^2+b^2}}\right)^8}$$

$$= \frac{a(a^2+b^2)^4}{b^8(a^2+b^2)^{1/2}} + \frac{b(a^2+b^2)^4}{a^8(a^2+b^2)^{1/2}}$$

$$= \pm\frac{(a^2+b^2)^4}{\sqrt{a^2+b^2}}\left(\frac{a}{b^8} + \frac{b}{a^8}\right).$$

42. (c) Given that $a\cos\theta + b\sin\theta = m$

and $a\sin\theta - b\cos\theta = n$.

Squaring and adding, we get

$(a\cos\theta + b\sin\theta)^2 + (a\sin\theta - b\cos\theta)^2 = m^2 + n^2$

$\Rightarrow a^2(\cos^2\theta + \sin^2\theta) + b^2(\cos^2\theta + \sin^2\theta)$

$+ 2ab(\cos\theta\sin\theta - \sin\theta\cos\theta) = m^2 + n^2$

Hence, $a^2 + b^2 = m^2 + n^2$.

Trick : Here we can guess that the value of $a^2 + b^2$ is independent of θ, so put any suitable value of θ i.e. $\frac{\pi}{2}$, so that $b = m$ and $a = n$. Hence $a^2 + b^2 = m^2 + n^2$. (Also check for other value of θ).

43. (c) $\left(\frac{x}{a}\right)^{1/3} = \cos\theta$, $\left(\frac{y}{b}\right)^{1/3} = \sin\theta$

Now square and add.

44. (a) As given

$\frac{1}{\tan\theta} + \tan\theta = m \Rightarrow 1 + \tan^2\theta = m\tan\theta$

$\Rightarrow \sec^2\theta = m\tan\theta$ .....(i)

and $\sec\theta - \cos\theta = n \Rightarrow \sec^2\theta - 1 = n\sec\theta$

$\Rightarrow \tan^2\theta = n\sec\theta$

$\Rightarrow \tan^4\theta = n^2\sec^2\theta = n^2.m\tan\theta$ {by (i)}

$\Rightarrow \tan^3\theta = n^2 m$, ($\because \tan\theta \neq 0$)

$\Rightarrow \tan\theta = (n^2 m)^{1/3}$ .....(ii)

Also, $\sec^2\theta = m\tan\theta = m(n^2 m)^{1/3}$ {by (i) and (ii)}

$\therefore$ Using the identity $\sec^2\theta - \tan^2\theta = 1$

$\Rightarrow m(mn^2)^{1/3} - (n^2 m)^{2/3} = 1$

$\Rightarrow m(mn^2)^{1/3} - n(nm^2)^{1/3} = 1.$

45. (c) $\sin^6\theta + \cos^6\theta + 3\sin^2\theta\cos^2\theta$

$= (\sin^2\theta + \cos^2\theta)^3 - 3\sin^2\theta\cos^2\theta + 3\sin^2\theta\cos^2\theta = 1.$

Trick : Put $\theta = 0^o$, we get the value of expression equal to 1. Again put $\theta = 45^o$, the value remains 1, it means that the expression is independent of θ and is equal to 1.

46. (b) $(\sin^2\theta + \cos^2\theta)^3 = (1)^3$

$\Rightarrow \sin^6\theta + \cos^6\theta + 3\sin^2\theta\cos^2\theta = 1$

and $\sin^4\theta + \cos^4\theta + 2\sin^2\theta\cos^2\theta = 1$

Both gives,

$2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1 = 0$.

47. (c) We have, $\sin x + \sin^2 x = 1$

or $\sin x = 1 - \sin^2 x$ or $\sin x = \cos^2 x$

$\therefore\ \cos^{12}x + 3\cos^{10}x + 3\cos^8 x + \cos^6 x - 2$

$= \sin^6 x + 3\sin^5 x + 3\sin^4 x + \sin^3 x - 2$

$= (\sin^2 x)^3 + 3(\sin^2 x)^2\sin x$

$+ 3(\sin^2 x)(\sin x)^2 + (\sin x)^3 - 2$

$= (\sin^2 x + \sin x)^3 - 2 = (1)^3 - 2$

[$\because \sin x + \sin^2 x = 1$(given)]

$= -1$.

48. (a) $\cos x + \cos^2 x = 1 \Rightarrow \cos x = \sin^2 x$

$\therefore \sin^2 x + \sin^4 x = \cos x + \cos^2 x = 1$.

49. (d) We have $\sin x + \sin^2 x = 1 \Rightarrow \sin x = \cos^2 x$

$\therefore$

$\cos^8 x + 2\cos^6 x + \cos^4 x = \sin^4 x + 2\sin^3 x + \sin^2 x$

$= (\sin x + \sin^2 x)^2 = 1$.

50. (c) We have $x\sin^3\alpha + y\cos^3\alpha = \sin\alpha\cos\alpha$ .....(i)

and $x\sin\alpha - y\cos\alpha = 0$ .....(ii)

Now from (ii), $x\sin\alpha = y\cos\alpha$

Putting in (i), we get

$\Rightarrow y\cos\alpha\sin^2\alpha + y\cos^3\alpha = \sin\alpha\cos\alpha$

$\Rightarrow y\cos\alpha\{\sin^2\alpha + \cos^2\alpha\} = \sin\alpha\cos\alpha$

$\Rightarrow y\cos\alpha = \sin\alpha\cos\alpha \Rightarrow y = \sin\alpha$ and $x = \cos\alpha$

Hence, $x^2 + y^2 = \sin^2\alpha + \cos^2\alpha = 1$.

51. (b) Multiplying both sides by $(1-\sin A)(1-\sin B)(1-\sin C)$,

we have, $(1-\sin^2 A)(1-\sin^2 B)(1-\sin^2 C)$

$$= (1-\sin A)^2(1-\sin B)^2(1-\sin C)^2$$

$\Rightarrow (1-\sin A)(1-\sin B)(1-\sin C) = \pm\cos A\cos B\cos C$

Similarly, $(1+\sin A)(1+\sin B)(1+\sin C) = \pm\cos A\cos B\cos C$.

52. (a) Given : $(\sec\alpha+\tan\alpha)(\sec\beta+\tan\beta)(\sec\gamma+\tan\gamma) = \tan\alpha\tan\beta\tan\gamma$ ...(i)

Let $x = (\sec\alpha-\tan\alpha)(\sec\beta-\tan\beta)(\sec\gamma-\tan\gamma)$ ...(ii)

Multiply both equations, (i) and (ii), we get

$(\sec^2\alpha-\tan^2\alpha)(\sec^2\beta-\tan^2\beta)(\sec^2\gamma-\tan^2\gamma) = x.(\tan\alpha\tan\beta\tan\gamma)$

$\Rightarrow x = \dfrac{1}{\tan\alpha\tan\beta\tan\gamma}$ $\quad\therefore x = \cot\alpha\cot\beta\cot\gamma$

53. (d) $\sin\theta_1 + \sin\theta_2 + \sin\theta_3 = 3$

$\Rightarrow \sin\theta_1 = \sin\theta_2 = \sin\theta_3 = 1$,

$(\because -1 \le \sin x \le 1)$

$\Rightarrow \theta_1 = \theta_2 = \theta_3 = \dfrac{\pi}{2} \Rightarrow \cos\theta_1 + \cos\theta_2 + \cos\theta_3 = 0$.

54. (d) $\sin^2\theta \le 1$

$\therefore \dfrac{x^2+y^2+1}{2x} \le 1$

$x^2 + y^2 - 2x + 1 \le 0$.

$(x-1)^2 + y^2 \le 0$

It is possible, iff $x = 1$ and $y = 0$,

i.e., It also depends on value of y.

Hence, option (d) is correct.

55. (d) Given that $\tan\theta - \cot\theta = a$ .....(i)

and $\sin\theta + \cos\theta = b$ .....(ii)

Now $(b^2-1)^2(a^2+4)$

$= \{(\sin\theta+\cos\theta)^2-1\}^2\{(\tan\theta-\cot\theta)^2+4\}$

$= [1+\sin 2\theta - 1]^2[\tan^2\theta + \cot^2\theta - 2 + 4]$

$= \sin^2 2\theta(\text{cosec}^2\theta + \sec^2\theta)$

$= 4\sin^2\theta\cos^2\theta\left[\dfrac{1}{\sin^2\theta} + \dfrac{1}{\cos^2\theta}\right] = 4$.

Trick : Obviously the value of expression $(b^2-1)^2(a^2+4)$ is independent of θ, therefore put any suitable value of θ. Let $\theta = 45°$, we get $a = 0, b = \sqrt{2}$ so that $[(\sqrt{2})^2-1]^2\ (0^2+4) = 4$.

56. (c) $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$

$= \dfrac{\tan^2\alpha}{1+\tan^2\alpha} + \dfrac{\tan^2\beta}{1+\tan^2\beta} + \dfrac{\tan^2\gamma}{1+\tan^2\gamma}$

$= \dfrac{x}{1+x} + \dfrac{y}{1+y} + \dfrac{z}{1+z}$

$(x = \tan^2\alpha, y = \tan^2\beta, z = \tan^2\gamma)$

$= \dfrac{(x+y+z)+(xy+yz+zx+2xyz)+xy+yz+zx+xyz}{(1+x)(1+y)(1+z)}$

$= \dfrac{1+x+y+z+xy+yz+zx+xyz}{(1+x)(1+y)(1+z)} = 1$

$(\because xy+yz+zx+2xyz = 1)$

57. (a) We know that one of the factor of the given expression is $\cos 90° = 0$.

Therefore $\cos 1°.\cos 2°.\cos 3°...\cos 179° = 0$.

58. (a) $\tan(90°-\theta) = \cot\theta, \cot(90°-\theta) = \tan\theta.$

Therefore $\dfrac{\cot 54°}{\tan 36°} + \dfrac{\tan 20°}{\cot 70°}$

$= \dfrac{\cot 54°}{\tan(90°-54°)} + \dfrac{\tan 20°}{(\cot 90°-20°)}$

$\dfrac{\cot 54°}{\cot 54°} + \dfrac{\tan 20°}{\tan 20°} = 1 + 1 = 2$.

59. (b) Since $\sin 190° = -\sin 10°, \sin 200° = -\sin 20°,$ $\sin 210° = -\sin 30°, \sin 360° = \sin 180° = 0$ etc.

60. (c) $(\cos 1° + \cos 179°) + (\cos 2° + \cos 178°) + ...$

$+(\cos 89° + \cos 91°) + \cos 90° + \cos 180° = -1$.

61. (a) We know, $\sin 22\dfrac{1^o}{2} = \dfrac{1}{2}\sqrt{2-\sqrt{2}}$

and $\cos 22\dfrac{1^o}{2} = \dfrac{1}{2}\sqrt{2+\sqrt{2}}$

$\therefore \left(1+\cos 22\dfrac{1^o}{2}\right)\left(1+\cos 67\dfrac{1^o}{2}\right)\left(1+\cos 112\dfrac{1^o}{2}\right)\left(1+\cos 157\dfrac{1^o}{2}\right)$

$= \left(1+\dfrac{1}{2}\sqrt{2+\sqrt{2}}\right)\left(1+\dfrac{1}{2}\sqrt{2-\sqrt{2}}\right)\left(1-\dfrac{1}{2}\sqrt{2-\sqrt{2}}\right)\left(1-\dfrac{1}{2}\sqrt{2+\sqrt{2}}\right)$

$= \left[1-\dfrac{1}{4}(2+\sqrt{2})\right]\left[1-\dfrac{1}{4}(2-\sqrt{2})\right]$

$= \dfrac{(4-2-\sqrt{2})(4-2+\sqrt{2})}{16}$

$= \dfrac{(2-\sqrt{2})(2+\sqrt{2})}{16} = \dfrac{4-2}{16} = \dfrac{1}{8}$.

62. (c) $6(\sin^6\theta+\cos^6\theta)-9(\sin^4\theta+\cos^4\theta)+4$

$=6[(\sin^2\theta+\cos^2\theta)^3-3\sin^2\theta\cos^2\theta(\sin^2\theta+\cos^2\theta)]$

$-9[(\sin^2\theta+\cos^2\theta)^2-2\sin^2\theta\cos^2]+4$

$=6[1-3\sin^2\theta\cos^2\theta]-9[1-2\sin^2\theta\cos^2\theta]+4$

$=6-9+4=1$.

63. (a) $\sin 15^\circ+\cos 105^\circ$

$\sin 15^\circ+\cos(90^\circ+15^\circ)=\sin 15^\circ-\sin 15^\circ=0$.

64. (d)

$\cos 105^\circ+\sin 105^\circ=\cos(90^\circ+15^\circ)+\sin(90^\circ+15^\circ)$

$=$

$\cos 15^\circ-\sin 15^\circ=\frac{\sqrt{3}+1}{2\sqrt{2}}-\frac{\sqrt{3}-1}{2\sqrt{2}}=\frac{2}{2\sqrt{2}}=\frac{1}{\sqrt{2}}$.

65. (d) The expression is equal to

$\sin(x-y)+\cos(x-y)=\sqrt{2}\left\{\sin\left(\frac{\pi}{4}+x-y\right)\right\}$,

which is zero, if $\sin\left(\frac{\pi}{4}+x-y\right)=0$

i.e., $\frac{\pi}{4}+x-y=n\pi(n\in I)\Rightarrow x=n\pi-\frac{\pi}{4}+y$.

66. (c) $\sin\frac{\pi}{10}\sin\frac{3\pi}{10}=\sin 18^\circ.\sin 54^\circ$

$=\sin 18^\circ.\cos 36^\circ=\frac{\sqrt{5}-1}{4}.\frac{\sqrt{5}+1}{4}=\frac{1}{4}$.

67. (c) $x.\frac{1}{\sqrt{2}}.\frac{1}{4}=\frac{3.2}{\sqrt{2}.3}\Rightarrow\frac{x}{4\sqrt{2}}=\sqrt{2}\Rightarrow x=8$.

68. (a) $x=\cos 40^\circ+\cos 130^\circ=2\cos 85^\circ\cos 45^\circ>0$.

69. (b)

$\cos A+\sin(270^\circ+A)-\sin(270^\circ-A)+\cos(180^\circ+A)$

$=\cos A-\cos A+\cos A-\cos A=0$.

70. (b) $\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}+\sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}}=\frac{1-\cos\alpha+1+\cos\alpha}{\sqrt{1-\cos^2\alpha}}$

$=\frac{2}{\pm\sin\alpha}=\frac{2}{-\sin\alpha}$, $\left(\text{since }\pi<\alpha<\frac{3\pi}{2}\right)$.

71. (a) $\tan\left(\frac{\pi}{4}+\theta\right)-\tan\left(\frac{\pi}{4}-\theta\right)=\frac{1+\tan\theta}{1-\tan\theta}-\frac{1-\tan\theta}{1+\tan\theta}$

$=\frac{4\tan\theta}{1-\tan^2\theta}=2\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)=2\tan 2\theta$.

72. (b) $\sin(\pi+\theta)\sin(\pi-\theta)\text{cosec}^2\theta$

$=-\sin\theta\sin\theta\frac{1}{\sin^2\theta}=-1$.

73. (c)

$\cot(45^\circ+\theta)\cot(45^\circ-\theta)=\tan(90^\circ-45^\circ-\theta)\cot(45^\circ-\theta)$

$=\tan(45^\circ-\theta)\cot(45^\circ-\theta)=1$.

74. (a)

$\tan A+\cot(180^\circ+A)+\cot(90^\circ+A)+\cot(360^\circ-A)$

$=\tan A+\cot A-\tan A-\cot A=0$.

75. (d) $\tan\theta\cos\theta\sin\theta=\sin^2\theta$.

76. (a) Let $A+B=\theta$ and $A-B=\phi$.

Then $\tan A=k\tan B$ or $\frac{k}{1}=\frac{\tan A}{\tan B}=\frac{\sin A\cos B}{\cos A\sin B}$

Applying componendo and dividendo

$\Rightarrow\frac{k+1}{k-1}=\frac{\sin A\cos B+\cos A\sin B}{\sin A\cos B-\cos A\sin B}$

$=\frac{\sin(A+B)}{\sin(A-B)}=\frac{\sin\theta}{\sin\phi}\Rightarrow\sin\theta=\frac{k+1}{k-1}\sin\phi$.

77. (b) We have $x=y\cos\frac{2\pi}{3}=z\cos\frac{4\pi}{3}$

$\Rightarrow\frac{x}{1}=\frac{y}{-2}=\frac{z}{-2}=\lambda$ (say)

$\Rightarrow x=\lambda, y=-2\lambda, z=-2\lambda$

$\therefore xy+yz+zx=-2\lambda^2+4\lambda^2-2\lambda^2=0$.

78. (a,c) Given that $\pi<\alpha<\frac{3\pi}{2}$ *i.e.*, α is in third quadrant.

Now, $\sqrt{(4\sin^4\alpha+\sin^2 2\alpha)}+4\cos^2\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)$

$=\sqrt{(4\sin^4\alpha+4\sin^2\alpha\cos^2\alpha)}+2.2\cos^2\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)$

$=\sqrt{4\sin^2\alpha(\sin^2\alpha+\cos^2\alpha)}+2\left[1+\cos\left(\frac{\pi}{2}-\alpha\right)\right]$

$=\pm 2\sin\alpha+2+2\sin\alpha$

On taking $-ve$, answer is 2 and on taking $+ve$, answer is $2+4\sin\alpha$

But $\pi<\alpha<\frac{3\pi}{4}$, Hence answer is $2-4\sin\alpha$

because $\sin\alpha$ is $-ve$ in third quadrant.

79. (d) $\sin^2\frac{\pi}{8}+\sin^2\frac{3\pi}{8}+\sin^2\frac{5\pi}{8}+\sin^2\frac{7\pi}{8}$

$=\sin^2\frac{\pi}{8}+\sin^2\frac{3\pi}{8}+\sin^2\frac{3\pi}{8}+\sin^2\frac{\pi}{8}$

$=2\left(\sin^2\frac{\pi}{8}+\sin^2\frac{3\pi}{8}\right)=2\times 1=2$.

80. (c) $(\sec A+\tan A-1)(\sec A-\tan A+1)-2\tan A$

$=(\sec^2 A-\tan^2 A)+\sec A+\tan A-\sec A$

$+\tan A-1-2\tan A=0$

$(\because\sec^2 A-\tan^2 A=1)$

81. (a) $\tan(-945^\circ)=\tan[-(945^\circ)]$

$=-\tan[(2\times 360^\circ+225^\circ)]$

$=-\tan[225^\circ]=-\tan 45^\circ=-1$.

82. (b) $A+B=45^\circ$, therefore $2A=90^\circ-2B$

$\therefore\cos 2A=\sin 2B$.

83. (c) $\cos A-\sin A=\cos\frac{5\pi}{4}-\sin\frac{5\pi}{4},\left(\because A=\frac{5\pi}{4}\right)$

$=-\cos\frac{\pi}{4}+\sin\frac{\pi}{4}=-\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}=0$.

84. (d) $\cos(270+\theta)\cos(90-\theta)-\sin(270-\theta)\cos\theta$
$=\sin\theta.\sin\theta+\cos\theta.\cos\theta=1$.

85. (d) $-2\pi<\alpha-\beta<2\pi$
$\cos(\alpha-\beta)=1\Rightarrow\ \alpha-\beta=0\Rightarrow\ \alpha=\beta$

$\cos 2\alpha=\frac{1}{e}$ and $-2\pi<2\alpha<2\pi$

Hence, there will be four solutions.