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(1) $\log_{2\sqrt{2}} 32\sqrt[5]{4} = ?$ (Let it be x)

$\Rightarrow (2\sqrt{2})^x = 32\, 4^{1/5}$

$\Rightarrow (2^{3/2})^x = 2^5 \cdot 2^{2/5}$

$\Rightarrow 2^{3x/2} = 2^{27/5}$

$\Rightarrow \frac{3x}{2} = \frac{27}{5} \Rightarrow \boxed{x = \frac{18}{5}} = 3.6$ option(A)

(2) $\log_e\left(\frac{a+b}{2}\right) = \frac{1}{2}(\log_e a + \log_e b)$

$\Rightarrow \log\left(\frac{a+b}{2}\right) = \frac{1}{2}\log(ab)$ [we are trying to get $\log X = \log Y$]

$\Rightarrow \frac{a+b}{2} = \sqrt{ab}$

$\Rightarrow (a+b)^2 = 4ab \Rightarrow (a-b)^2 = 0$

$\Rightarrow \boxed{a = b}$ option A

(3) $\log_3 4 \cdot \log_4 5 \cdot \log_5 6 \cdot \log_6 7 \cdot \log_7 8 \cdot \log_8 9$

(different bases, hint - base changing Prop)

So whole expression $= \frac{\cancel{\log 4}}{\log 3} \cdot \frac{\cancel{\log 5}}{\cancel{\log 4}} \cdot \frac{\cancel{\log 6}}{\cancel{\log 5}} \cdot \frac{\cancel{\log 7}}{\cancel{\log 6}} \cdot \frac{\cancel{\log 8}}{\cancel{\log 7}} \cdot \frac{\log 9}{\cancel{\log 8}}$

$= \frac{\log 9}{\log 3}$

$= \log_3 9 = 2$ option(B)

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(4) $\log_7 \log_7 \sqrt{7\sqrt{7\sqrt{7}}}$

$= \log_7 \log_7 \left(\left(7^{1/2} \times 7\right)^{1/2} \times 7\right)^{1/2}$

$= \log_7 \log_7 \left[\left(7^{3/2}\right)^{1/2} \times 7\right]^{1/2}$

$= \log_7 \log_7 \left[7^{3/4+1}\right]^{1/2}$

$= \log_7 \log_7 7^{7/8}$

$= \log_7 \left(\frac{7}{8}\right)$ $\quad$ $\log_7 7^{7/8} = 7/8$

$= \log_7 7 - \log_7 2^3$

$= 1 - 3\log_7 2$ $\quad$ $\boxed{\text{option C}}$

(5) using $\log M^{\alpha} = \alpha \log M$, $\log MN = \log M + \log N$

$7\log\left(\frac{16}{15}\right) + 5\log\left(\frac{25}{24}\right) + 3\log\frac{81}{80}$

$= \log\left[\left(\frac{16}{15}\right)^7 \times \left(\frac{25}{24}\right)^5 \times \left(\frac{81}{80}\right)^3\right]$

$= \log\left[\frac{2^{\cancel{28}}}{\cancel{3^7}\,\cancel{5^7}} \times \frac{\cancel{5^{10}}}{\cancel{2^{15}}\,\cancel{3^5}} \times \frac{\cancel{3^{12}}}{\cancel{2^{12}}\,\cancel{5^3}}\right]$

$= \log[2] = \log 2$ $\quad$ $\boxed{\text{option C}}$

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Q.6 $\log_4 5 = a$, $\log_5 6 = b$ than $\log_3 2$

$\Rightarrow \cancel{4^a = 5 \Rightarrow 2^{2a} = 5,\ 5^b = 6}$

we want $\log_3 2$ in terms of a & b

$\dfrac{\log 5}{\log 4} = a$, $\dfrac{\log 6}{\log 5} = b$

multiplying above equations

$\dfrac{\log 6}{\log 4} = ab$

$\Rightarrow \dfrac{\log 2 + \log 3}{2\log 2} = ab$

$\Rightarrow \dfrac{1}{2} + \dfrac{1}{2}\log_2 3 = ab$

$\Rightarrow \log_2 3 = \dfrac{2ab-1}{1}$

$\Rightarrow \log_3 2 = \dfrac{1}{2ab-1}$ option(d)

Q.7 $\log_k x \log_5 k = \log_x 5$ $x = ?$

$\Rightarrow \dfrac{\log x}{\cancel{\log k}} \times \dfrac{\cancel{\log k}}{\log 5} = \dfrac{\log 5}{\log x}$

$\Rightarrow \dfrac{\log x}{\log 5} = \dfrac{\log 5}{\log x}$

$\Rightarrow \log x = \pm \log 5$

$\Rightarrow x = 5 \text{ or } 1/5$ B & C both

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Q.8 $a^2+4b^2=12ab$ $\log(a+2b) = ?$

means we need expression of type $a+2b$

$\Rightarrow a^2-12ab+4b^2=0$

$\Rightarrow a^2+4ab+4b^2=16ab$

$\Rightarrow (a+2b)^2 = 16ab$

$\Rightarrow \log(a+2b)^2 = \log 16ab$

$\Rightarrow 2\log(a+2b) = \log(16ab)$

$\Rightarrow \log(a+2b) = \frac{1}{2}[\log 2^4 + \log a + \log b]$

$\Rightarrow \log(a+2b) = 2\log 2 + \frac{1}{2}(\log a + \log b)$

$= \frac{1}{2}[\log a + \log b + 4\log 2]$

option c

Q.9 $\log_{10} x = y$ then $\log_{1000} x^2$ in terms of y

$\Rightarrow \cancel{\log}\ x = 10^y$ (since $\log_{10} x = y$)

Now

base changing

$\log_{1000} x^2 = \log_{1000} 10^{2y} = \frac{2y \log 10}{\log 10^3}$

$= \frac{2y \cancel{\log 10}}{3 \cancel{\log 10}}$

$= \frac{2y}{3}$

$\boxed{\text{option (d)}}$

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(10) $x = \log_a bc$, $y = \log_b ac$, $z = \log_c ab$.

we want to get an expression in x, y, z

$\Rightarrow$

So here $bc = a^x \Rightarrow a^{x+1} = abc$ (multiplied by a)

$ac = b^y \Rightarrow b^{y+1} = abc$

$ab = c^z \Rightarrow c^{z+1} = abc$

Now $a = (abc)^{\frac{1}{x+1}}$

$b = (abc)^{\frac{1}{y+1}}$

$c = (abc)^{\frac{1}{z+1}}$

Now $abc = (abc)^{\frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{z+1}}$

$\Rightarrow (x+1)^{-1} + (y+1)^{-1} + (z+1)^{-1} = 1$ Option B

** think : why did I solve like this.

(11) $a = \log_{24} 12$, $b = \log_{36} 24$, $c = \log_{48} 36$

$1 + abc = ?$ (interms of a, b & c)

$abc = \log_{24} 12 \cdot \log_{36} 24 \cdot \log_{48} 36$

$= \frac{\log 12}{\cancel{\log 24}} \times \frac{\cancel{\log 24}}{\cancel{\log 36}} \times \frac{\cancel{\log 36}}{\log 48}$

$abc = \log_{48} 12 = \frac{\log 12}{\log 48}$

$1 + abc = \frac{\log 48 + \log 12}{\log 48} = \frac{\log 12^2 \times 4}{\log 12 \times 4}$

$= \frac{\log 48 + \log 12}{\log 48}$

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Q.11 $a = \log_{24} 12$, $b = \log_{36} 24$, $c = \log_{48} 36$

$abc + 1 = \frac{\log 12}{\cancel{\log 24}} \times \frac{\cancel{\log 24}}{\cancel{\log 36}} \times \frac{\cancel{\log 36}}{\log 48} + 1$

$\frac{\log 12}{\log 48} + 1$

$= \frac{\log 12 + \log 48}{\log 48}$

This question is not straight ~~forward~~, look the options and think why ~~am~~ I am doing so we ~~wa~~ want to convert the abc + 1 into 2bc or ~~2ab~~ or 2ac etc

$\frac{\log 12 + \log 48}{\log 48} = \frac{\log (12 \times 48)}{\log 48} = \frac{\log 24^2}{\log 48}$

$= \frac{2 \log 24}{\log 48}$

$= 2 \cdot \frac{\log 24}{\log 36} \cdot \frac{\log 36}{\log 48}$

$= 2 \cdot bc$

* think again.

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(12) $\log_{10} 2 = 0.30103$; $\log_{10} 3 = 0.47712$

number of digits in $3^{12} \times 2^{8}$

$$N = 3^{12} \times 2^{8}$$

$$\log_{10} N = 12 \log_{10} 3 + 8 \log_{10} 2$$ [taking log at base 10]

$$\Rightarrow \log_{10} N = 12(0.47712) + 8(0.30103)$$

$$\log_{10} N = 8.13368$$

$$\Rightarrow N = 10^{8.133668}$$

$$N = \text{b/w } 10^{8} \text{ and } 10^{9}$$

so no of digits in N = 9 (C)

(13)

$$\log_{\frac{1}{2}} (x^2 - 6x + 12) \geq -2$$

$$\Rightarrow 0 < x^2 - 6x + 12 \leq 4$$ [apply maths but base is < 1, so change sign of inequation]

$$\Rightarrow 0 < \underbrace{(x-3)^2 + 3}_{\text{it is already} \geq 3} \leq 4$$

$$\Rightarrow (x-3)^2 + 3 \leq 4$$

$$\Rightarrow (x-3)^2 \leq 1$$

$$\Rightarrow -1 \leq x - 3 \leq 1$$

$$\Rightarrow 2 \leq x \leq 4 \Rightarrow x \in [2, 4]$$ (B)

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(14) $\log_{0.04}(x-1) \geq \log_{0.2}(x-1)$

make bases equal $(0.2^2 = 0.04)$

$\Rightarrow \dfrac{\log_{0.2}(x-1)}{\log_{0.2} 0.04} \geq \log_{0.2}(x-1)$

$\Rightarrow \dfrac{\log_{0.2}(x-1)}{2} \geq \log_{0.2}(x-1)$

$\Rightarrow \dfrac{X}{2} \geq X$

$\Rightarrow 2X - X \leq 0$

$\Rightarrow X \leq 0$

$\Rightarrow \log_{0.2}(x-1) \leq 0$

$\Rightarrow x-1 \geq (0.2)^0$ [base < 1]

$\Rightarrow x-1 \geq 1$

$\Rightarrow x \geq 2$

$\Rightarrow x \in [2, \infty)$ (C)

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(15) $\log_{0.2}\left(\frac{x+2}{x}\right) \le 1$

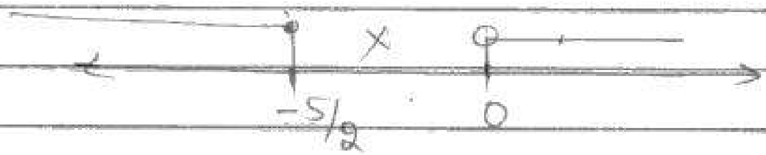
$\Rightarrow \frac{x+2}{x} \geq 0.2$ (Base <1)

$\Rightarrow \frac{x+2}{x} - \frac{1}{5} \geq 0$

$\Rightarrow \frac{5x+10-x}{5x} \geq 0$

$\Rightarrow \frac{4x+10}{5x} \geq 0$

$\Rightarrow \frac{2x+5}{5x} \geq 0$



$\Rightarrow x \in \left(-\infty, -\frac{5}{2}\right] \cup (0, \infty)$ (A)