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12th CBSE MATHS SET – 3 CODE 65/3/3

1. The value of $\begin{vmatrix} 8 & 2 & 7 \\ 12 & 3 & 5 \\ 16 & 4 & 3 \end{vmatrix}$ is :

A (A) 0 (B) 2
(C) 7 (D) – 2

2. If $y = \sin^{-1} x$, then $\frac{d^2y}{dx^2}$ is :

C (A) sec y (B) sec y tan y
(C) $\sec^2 y \tan y$ (D) $\tan^2 y \sec y$

3. If $|\vec{a}| = 2$ and $-3 \le k \le 2$, then $|k\vec{a}| \in$:

(A) [– 6, 4] (B) [0, 4]
D (C) [4, 6] (D) [0, 6]

4. If a line makes an angle of $\frac{\pi}{4}$ with the positive directions of both x-axis and z-axis, then the angle which it makes with the positive direction of y-axis is :

C (A) 0 (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) π

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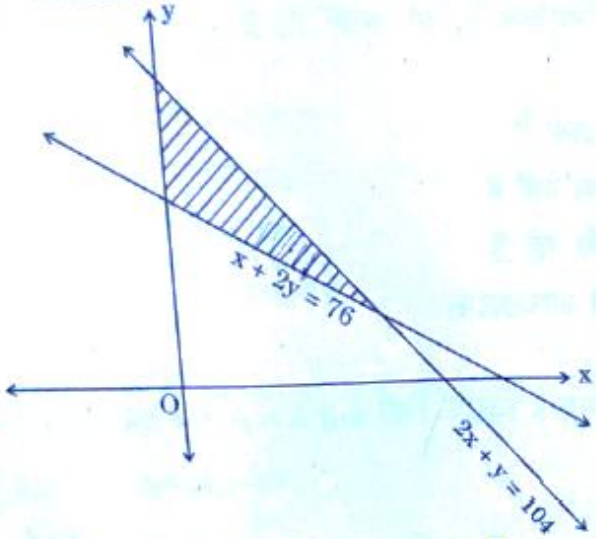
5. Of the following, which group of constraints represents the feasible region given below ?



C

(A) $x + 2y \leq 76, 2x + y \geq 104, x, y \geq 0$
(B) $x + 2y \leq 76, 2x + y \leq 104, x, y \geq 0$
(C) $x + 2y \geq 76, 2x + y \leq 104, x, y \geq 0$
(D) $x + 2y \geq 76, 2x + y \geq 104, x, y \geq 0$

6. If $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$, then A^{-1} is :

(A) $\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$

(B) $30\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$

A

(C) $\frac{1}{30}\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

(D) $\frac{1}{30}\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$

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7. If $A = [a_{ij}]$ is an identity matrix, then which of the following is true ?

D

(A) $a_{ij} = \begin{cases} 0, & \text{if } i = j \\ 1, & \text{if } i \neq j \end{cases}$ (B) $a_{ij} = 1, \forall\ i, j$

(C) $a_{ij} = 0, \forall\ i, j$ (D) $a_{ij} = \begin{cases} 0, & \text{if } i \neq j \\ 1, & \text{if } i = j \end{cases}$

8. Let Z denote the set of integers, then function $f : Z \to Z$ defined as $f(x) = x^3 - 1$ is :

B

(A) both one-one and onto

(B) one-one but not onto

(C) onto but not one-one

(D) neither one-one nor onto

9. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be a square matrix such that adj A = A. Then, $(a + b + c + d)$ is equal to :

A

(A) 2a (B) 2b

(C) 2c (D) 0

10. A function $f(x) = |1 - x + |x||$ is :

D

(A) discontinuous at x = 1 only (B) discontinuous at x = 0 only

(C) discontinuous at x = 0, 1 (D) continuous everywhere

11. The rate of change of surface area of a sphere with respect to its radius 'r', when r = 4 cm, is :

C

(A) 64π cm^2/cm (B) 48π cm^2/cm

(C) 32π cm^2/cm (D) 16π cm^2/cm

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12. $\int_{-a}^{a} f(x)\,dx = 0$, if :

B

(A) $f(-x) = f(x)$ (B) $f(-x) = -f(x)$

(C) $f(a-x) = f(x)$ (D) $f(a-x) = -f(x)$

13. $x \log x \frac{dy}{dx} + y = 2 \log x$ is an example of a :

C

(A) variable separable differential equation.

(B) homogeneous differential equation.

(C) first order linear differential equation.

(D) differential equation whose degree is not defined.

14. If $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + \hat{j} - \hat{k}$, then $\vec{a}$ and $\vec{b}$ are :

C

(A) collinear vectors which are not parallel

(B) parallel vectors

(C) perpendicular vectors

(D) unit vectors

15. If α, β and γ are the angles which a line makes with positive directions of x, y and z axes respectively, then which of the following is ***not*** true ?

D

(A) $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

(B) $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

(C) $\cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$

(D) $\cos\alpha + \cos\beta + \cos\gamma = 1$

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16. The restrictions imposed on decision variables involved in an objective function of a linear programming problem are called :

B (A) feasible solutions (B) constraints
(C) optimal solutions (D) infeasible solutions

17. Let E and F be two events such that P(E) = 0·1, P(F) = 0·3, P(E ∪ F) = 0·4, then P(F | E) is :

D (A) 0·6 (B) 0·4 (C) 0·5 (D) 0

18. If A and B are two skew symmetric matrices, then (AB + BA) is :

B (A) a skew symmetric matrix (B) a symmetric matrix
(C) a null matrix (D) an identity matrix

Questions number ***19*** *and* ***20*** *are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.*

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. *Assertion (A) :* For any non-zero unit vector $\vec{a}$, $\vec{a} \cdot (-\vec{a}) = (-\vec{a}) \cdot \vec{a} = -1$.

C *Reason (R) :* Angle between $\vec{a}$ and $(-\vec{a})$ is $\frac{\pi}{2}$.

20. *Assertion (A) :* Every scalar matrix is a diagonal matrix.

C *Reason (R) :* In a diagonal matrix, all the diagonal elements are 0.

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21. $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three mutually perpendicular unit vectors. If θ is the angle between $\vec{a}$ and $(2\vec{a} + 3\vec{b} + 6\vec{c})$, find the value of $\cos\theta$.

(21)

$a = b = c = 1$

$\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{c} = \vec{a}\cdot\vec{c} = 0$

$$\cos\theta = \frac{\vec{a}\cdot(2\vec{a} + 3\vec{b} + 6\vec{c})}{1\cdot|2\vec{a} + 3\vec{b} + 6\vec{c}|} = \cos\theta$$

$$\Rightarrow \cos\theta = \frac{2+0+0}{\sqrt{4+9+36}} = \frac{2}{7} \text{ Ans.}$$

$\cos\theta = \frac{2}{7}$

22. Evaluate :

$$\cot^2\left\{\text{cosec}^{-1}3\right\} + \sin^2\left\{\cos^{-1}\left(\frac{1}{3}\right)\right\}$$

(22)

$$\cot^2(\text{cosec}^{-1}3) + \sin^2(\cos^{-1}\tfrac{1}{3})$$

$$= \cot^2(\cot^{-1}\sqrt{8}) + \sin^2(\sin^{-1}\tfrac{\sqrt{8}}{3})$$

$$= (\sqrt{8})^2 + \left(\frac{\sqrt{8}}{3}\right)^2 = 8 + \frac{8}{9} = \frac{80}{9} \text{ Ans.}$$

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23. (a) If $x = e^{x/y}$, prove that $\frac{dy}{dx} = \frac{\log x - 1}{(\log x)^2}$

OR

(b) Check the differentiability of $f(x) = \begin{cases} x^2+1, & 0 \le x < 1 \\ 3-x, & 1 \le x \le 2 \end{cases}$ at $x = 1$.

(23) $x = e^{x/y}$ PT $\frac{dy}{dx} = \frac{\ln x - 1}{(\ln x)^2}$

$\Rightarrow \frac{x}{y} = \ln x$

$\Rightarrow y = \frac{x}{\ln x}$

$\Rightarrow \frac{dy}{dx} = \frac{\ln x - x \cdot \frac{1}{x}}{(\ln x)^2} = \frac{\ln x - 1}{(\ln x)^2}$ Hence Proved

OR

23 OR

$f(x) = \begin{cases} x^2+1 & 0 \le x < 1 \\ 3-x & 1 < x \le 2 \end{cases}$ check diff at $x = 1$

$f'(1^-) = \lim_{h \to 0} \frac{f(1-h) - f(1)}{-h}$

$= \lim_{h \to 0} \frac{(1-h)^2 + 1 - 2}{-h} = \lim_{h \to 0} \frac{(-2h + h^2)}{-h} = +2$

$f'(1^+) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$

$= \lim_{h \to 0} \frac{3-(1+h) - 2}{h} = \lim_{h \to 0} \frac{-h}{h} = -1$

Since $f'(1^-) \neq f'(1^+) \Rightarrow f$ is not diff at $x = 1$

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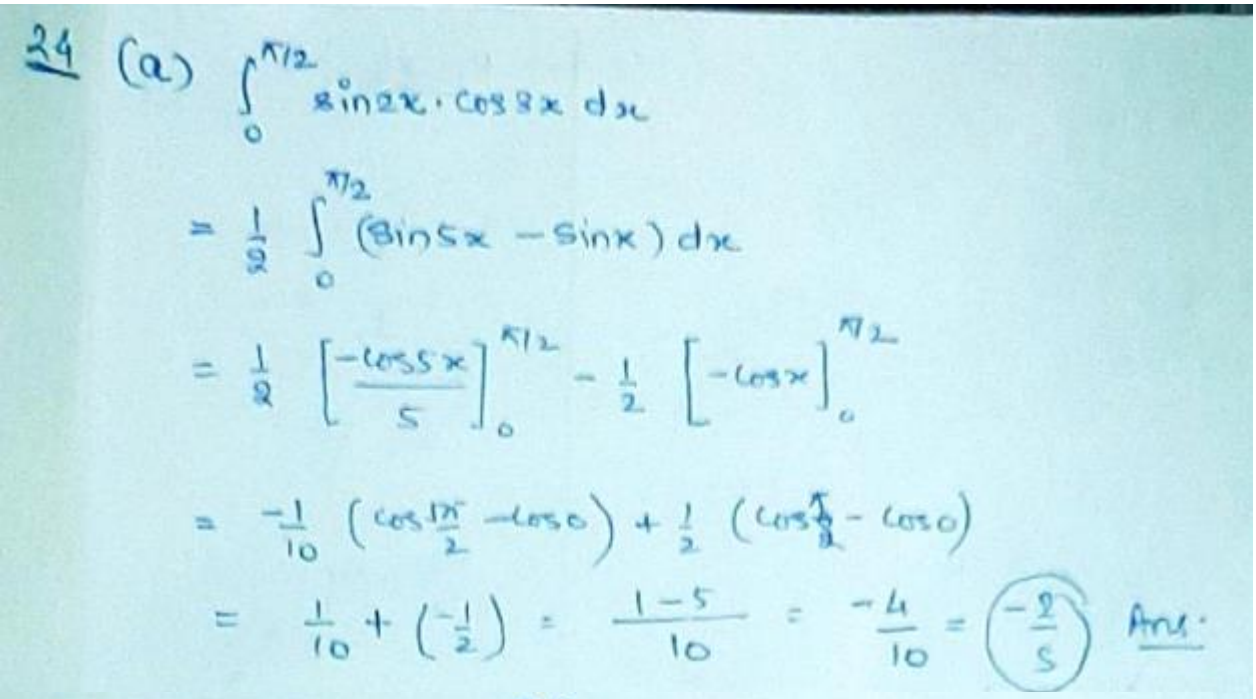
24. (a) Evaluate :

$$\int_0^{\pi/2} \sin 2x \cos 3x \, dx$$

OR

(b) Given $\frac{d}{dx} F(x) = \frac{1}{\sqrt{2x - x^2}}$ and $F(1) = 0$, find $F(x)$.



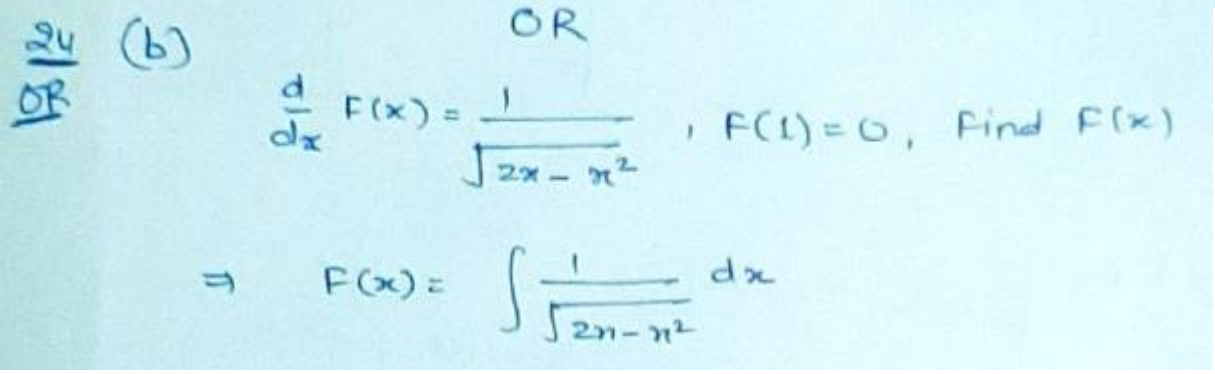
24 (a) $\int_0^{\pi/2} \sin 2x \cdot \cos 3x \, dx$

$= \frac{1}{2} \int_0^{\pi/2} (\sin 5x - \sin x) \, dx$

$= \frac{1}{2} \left[\frac{-\cos 5x}{5}\right]_0^{\pi/2} - \frac{1}{2} \left[-\cos x\right]_0^{\pi/2}$

$= \frac{-1}{10} \left(\cos \frac{5\pi}{2} - \cos 0\right) + \frac{1}{2} \left(\cos \frac{\pi}{2} - \cos 0\right)$

$= \frac{1}{10} + \left(\frac{-1}{2}\right) = \frac{1-5}{10} = \frac{-4}{10} = \left(\frac{-2}{5}\right)$ Ans.



24 OR (b)

OR

$\frac{d}{dx} F(x) = \frac{1}{\sqrt{2x - x^2}}$, $F(1) = 0$, Find $F(x)$

$\Rightarrow F(x) = \int \frac{1}{\sqrt{2x - x^2}} \, dx$

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$$= \int \frac{dx}{\sqrt{1-(x-1)^2}}$$

$$F(x) = \sin^{-1}(x-1) + C$$

$$F(1) = 0 = \sin^{-1} 0 + C \Rightarrow C = 0$$

$$\Rightarrow \boxed{F(x) = \sin^{-1}(x-1)}$$

25. Find the position vector of point C which divides the line segment joining points A and B having position vectors $\hat{i} + 2\hat{j} - \hat{k}$ and $-\hat{i} + \hat{j} + \hat{k}$ respectively in the ratio 4 : 1 externally. Further, find $|\vec{AB}| : |\vec{BC}|$.

$\vec{OC} = ?$

$-4 \quad 1$

A $(1,2,-1)$ C B $(-1,1,1)$ $\quad \frac{AB}{BC} = ?$

$$\vec{OC} = \frac{-4\vec{OB} + \vec{OA}}{-3} = \frac{4\hat{i} - 4\hat{j} - 4\hat{k} + \hat{i} + 2\hat{j} - \hat{k}}{-3}$$

$$= \frac{5\hat{i} - 2\hat{j} - 5\hat{k}}{-3} = \frac{-5\hat{i} + 2\hat{j} + 5\hat{k}}{3}$$

P.V. of C, $\vec{OC} = \frac{-5\hat{i} + 2\hat{j} + 5\hat{k}}{3}$

$\leftarrow 4\lambda \rightarrow$

A B C $\quad \lambda$

$\Rightarrow \frac{AB}{BC} = \frac{4\lambda}{\lambda} = \frac{4}{1}$

(4 : 1)

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26. Solve the following linear programming problem graphically :

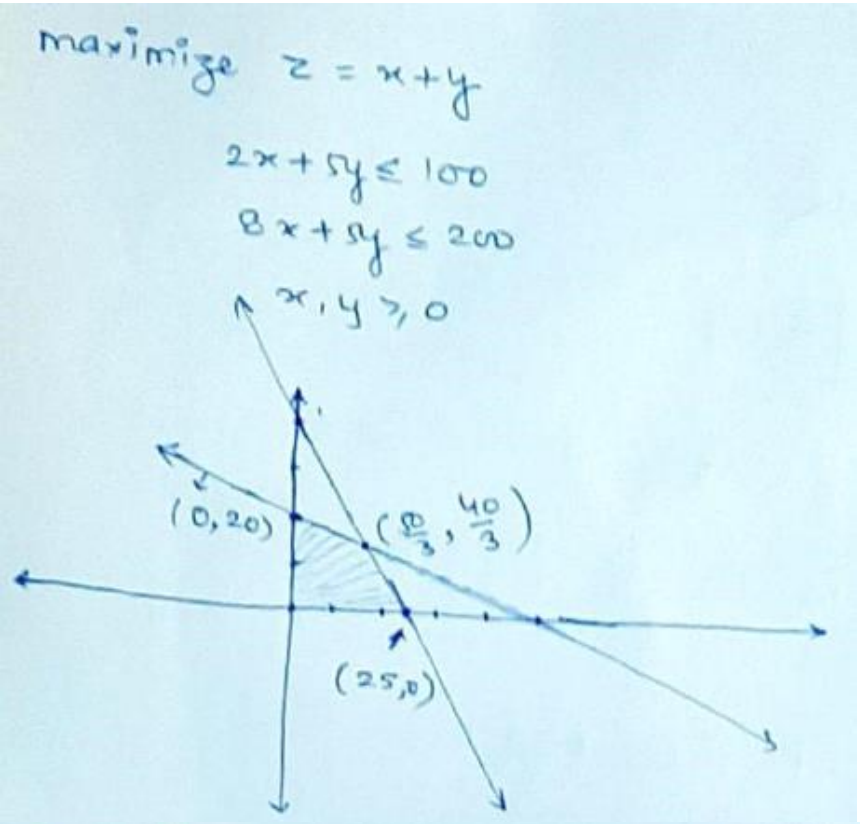
Maximize $z = x + y$

subject to constraints

$2x + 5y \leq 100$

$8x + 5y \leq 200$

$x \geq 0, y \geq 0.$



Corner Point	$z = x + y$
(0, 0)	0
(25, 0)	25
(0, 20)	20
$\left(\frac{50}{3}, \frac{40}{3}\right)$	30 ← max

Ans.

$z_{max} = 30$ at $\left(\frac{50}{3}, \frac{40}{3}\right)$

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27. The chances of P, Q and R getting selected as CEO of a company are in the ratio 4 : 1 : 2 respectively. The probabilities for the company to increase its profits from the previous year under the new CEO, P, Q or R are 0·3, 0·8 and 0·5 respectively. If the company increased the profits from the previous year, find the probability that it is due to the appointment of R as CEO.

$P(P \text{ is selected}) = \frac{4K}{7}$

$P(Q \text{ ——— }) = \frac{K}{7}$

$P(R \text{ ——— }) = \frac{2K}{7}$

$P(\text{profit increased}/P \text{ is selected}) = 0.3$

$P(\text{ ——— }/Q \text{ ——— }) = 0.8$

$P(\text{ ——— }/R \text{ ——— }) = 0.5$

$P(\text{Profit is increased}) = 4K \times 0.3 + K \times 0.8 + 2K \times 0.5$

$= 1.2K + 0.8K + K = 3K$

$P(R \text{ is selected}/\text{Profit is } \uparrow) = \frac{P(R \text{ is selected \& profit } \uparrow)}{P(\text{Profit is } \uparrow)}$

$= \frac{2K(0.5)}{3K}$

$= \frac{1}{3}$ Ans.

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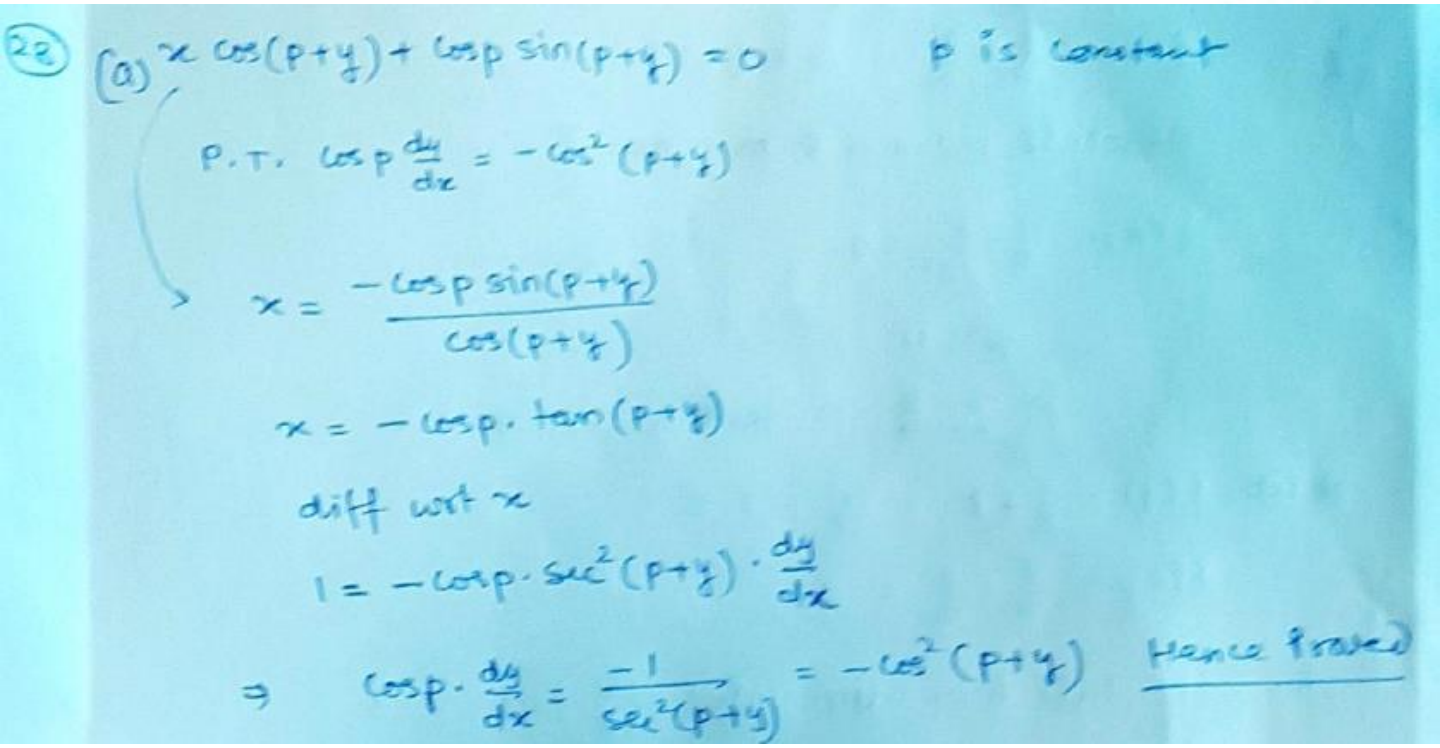
28. (a) If $x \cos(p + y) + \cos p \sin(p + y) = 0$, prove that $\cos p \dfrac{dy}{dx} = -\cos^2(p + y)$, where p is a constant.

OR

(b) Find the value of a and b so that function f defined as :

$$f(x) = \begin{cases} \dfrac{x-2}{|x-2|} + a, & \text{if } x < 2 \\ a + b, & \text{if } x = 2 \\ \dfrac{x-2}{|x-2|} + b, & \text{if } x > 2 \end{cases}$$

is a continuous function.



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OR

(b)

$$f(x) = \begin{cases} \frac{x-2}{|x-2|} + a & x < 2 \\ a+b & x = 2 \\ \frac{x-2}{|x-2|} + b & x > 2 \end{cases}$$

$a = ?$, $b = ?$

f is Continuous

$$\Rightarrow f(x) = \begin{cases} -1+a & x < 2 \\ a+b & x = 2 \\ 1+b & x > 2 \end{cases}$$

$|x-2| = -(x-2)$ if $x < 2$

$f(2^-) = f(2) = f(2^+)$ f is Continuous at $x = 2$

$\Rightarrow a - 1 = a + b = 1 + b$

$\Rightarrow$ $b = -1$, $a = 1$ Ans

29. (a) Find the intervals in which the function $f(x) = \frac{\log x}{x}$ is strictly increasing or strictly decreasing.

OR

(b) Find the absolute maximum and absolute minimum values of the function f given by $f(x) = \frac{x}{2} + \frac{2}{x}$, on the interval $[1, 2]$.

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(a) $f(x) = \frac{\ln x}{x}$

$f'(x) = \frac{x \cdot \frac{1}{x} - \ln x}{x^2} = \frac{1 - \ln x}{x^2}$

If f is ↑ then

$\Rightarrow f'(x) > 0$

$\frac{1 - \ln x}{x^2} > 0 \Rightarrow 1 - \ln x > 0$

$\ln x - 1 < 0$

$\ln x < 1$

$0 < x < e$

So f is increasing in $(0, e)$ and decreasing in (e, ∞)

Ans.

29 OR

OR

$f(x) = \frac{x}{2} + \frac{2}{x} \quad x \in [1, 2]$

absolute maxima & minima = ?

$f'(x) = \frac{1}{2} - \frac{2}{x^2} = 0$

$\Rightarrow x^2 = 4$

$\Rightarrow x = 2 \Rightarrow$ testing points 1, 2

No Sab $f(1) = \frac{1}{2} + 2 = 5/2 = 2.5$

$f(2) = \frac{2}{2} + \frac{2}{2} = 2$

So absolute maximum value = $\left(\frac{5}{2}\right)$ at $x = 1$

absolute minimum value = (2) at $x = 2$

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30. Find :

$$\int \frac{\sqrt{x}}{(x+1)(x-1)}\,dx$$

(30)

$$\int \frac{\sqrt{x}}{(x+1)(x-1)}\,dx$$

$x = t^2 \Rightarrow dx = 2t\,dt$

$$I = \int \frac{t\,(2t\,dt)}{(t^2+1)(t^2-1)} = \int \frac{2t^2}{(t^2+1)(t^2-1)}\,dt$$

$$= \int \left(\frac{1}{t^2-1} + \frac{1}{t^2+1}\right) dt$$

$$= \frac{1}{2}\ln\left(\frac{t-1}{t+1}\right) + \tan^{-1} t + C$$

$$\boxed{I = \frac{1}{2}\ln\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right) + \tan^{-1}\sqrt{x} + C}$$ Ans.

31. (a) Find :

$$\int \frac{2+\sin 2x}{1+\cos 2x}\,e^x\,dx$$

OR

(b) Evaluate :

$$\int_0^{\pi/4} \frac{1}{\sin x + \cos x}\,dx$$

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(31) (a) $\int \frac{2+\sin 2x}{1+\cos 2x} \cdot e^x dx$

$= \int e^x \left(\frac{2+2\sin x \cos x}{2\cos^2 x}\right)$

$= \int e^x (\sec^2 x + \tan x) dx$

$I = e^x \tan x + C$ Ans

(b) OR

$I = \int_0^{\pi/4} \frac{1}{\sin x + \cos x} dx$

$I = \int_0^{\pi/4} \frac{dx}{\sqrt{2}\sin(x+\pi/4)}$

$\int \text{cosec}\, x\, dx = \ln|\text{cosec}\, x - \cot x|$

$= \frac{1}{\sqrt{2}} \int_0^{\pi/4} \text{cosec}(x+\pi/4)\, dx$

$= \frac{1}{\sqrt{2}} \left[\log\left(\text{cosec}(x+\pi/4) - \cot(x+\pi/4)\right)\right]_0^{\pi/4}$

$= \frac{1}{\sqrt{2}} \left[\ln\left(\text{cosec}\frac{\pi}{2} - \cot\frac{\pi}{2}\right) - \ln\left(\text{cosec}\,\pi/4 - \cot\pi/4\right)\right]$

$= \frac{1}{\sqrt{2}} \left[\ln(1-0) - \ln(\sqrt{2}-1)\right]$

$= \frac{1}{\sqrt{2}} \left[\ln \frac{1}{\sqrt{2}-1}\right]$

$I = \frac{1}{\sqrt{2}} \ln(\sqrt{2}+1)$ Ans

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32. (a) Find the equation of the line passing through the point of intersection of the lines $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ and $\frac{x-1}{0} = \frac{y}{-3} = \frac{z-7}{2}$ and perpendicular to these given lines.

OR

(b) Two vertices of the parallelogram ABCD are given as A(–1, 2, 1) and B(1, –2, 5). If the equation of the line passing through C and D is $\frac{x-4}{1} = \frac{y+7}{-2} = \frac{z-8}{2}$, then find the distance between sides AB and CD. Hence, find the area of parallelogram ABCD.

given lines $L_1 : \frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3} = \lambda \Rightarrow$ drs = 1, 2, 3

$L_2 : \frac{x-1}{0} = \frac{y}{-3} = \frac{z-7}{2} = \mu \Rightarrow$ drs = 0, –3, 2

any Point on $L_1 = (\lambda, 2\lambda+1, 3\lambda+2)$

$L_2 = (1, -3\mu, 2\mu+7)$

Point of intersection $(\lambda, 2\lambda+1, 3\lambda+2) = (1, -3\mu, 2\mu+7)$

$\Rightarrow \lambda = 1$, $\mu = -1$

So point of intersection = (1, 3, 5)

drs of line ⊥ to L_1 & L_2 :

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 0 & -3 & 2 \end{vmatrix}$$

$= \hat{i}(4+9) - \hat{j}(2) + \hat{k}(-3)$

$= 13\hat{i} - 2\hat{j} - 3\hat{k} \equiv$ 13, –2, –3

So required line $\frac{x-1}{13} = \frac{y-3}{-2} = \frac{z-5}{-3}$ Ans

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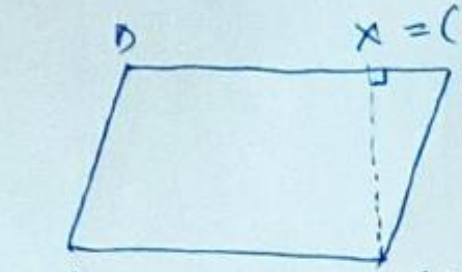
OR
b.



$X = (\lambda+4, -2\lambda-7, 2\lambda+8)$

Eq^n of line CD : $\frac{x-4}{1} = \frac{y+7}{-2} = \frac{z-8}{2} = \lambda$

distance b/w AB & CD = BX

$X = (\lambda+4, -2\lambda-7, 2\lambda+8)$

drs of BX = $\lambda+3, -2\lambda-5, 2\lambda+3$

drs of CD = $1, -2, 2$

$BX \perp CD \Rightarrow (\lambda+3) - 2(-2\lambda-5) + 2(2\lambda+3) = 0$

$\Rightarrow \lambda+3+4\lambda+10+4\lambda+6 = 0$

$\Rightarrow 9\lambda + 19 = 0$

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$$\lambda = \frac{-19}{9}$$

So $BX = \left(\frac{-19}{9}+4, \frac{38}{9}-7, \frac{-38}{9}+8\right)$

$$= \left(+\frac{17}{9}, -\frac{25}{9}, \frac{34}{9}\right)$$

$$BX = \sqrt{\left(\frac{-19}{9}+3\right)^2 + \left(\frac{38}{9}-5\right)^2 + \left(\frac{-38}{9}+3\right)^2}$$

$$= \sqrt{\frac{64}{81} + \frac{49}{81} + \frac{121}{81}}$$

$$BX = \sqrt{\frac{234}{81}} = \frac{\sqrt{234}}{9}$$ Ans.

Area of $\parallel gm = AB \times BX$

$$= 2 \times \frac{\sqrt{234}}{3}$$ Ans.

$$= \frac{2}{3}\sqrt{234} \text{ Sq. units}$$

AB =

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33. Let A = R – {3} and B = R – {a}. Find the value of 'a' such that the function f : A → B defined by $f(x) = \frac{x-2}{x-3}$ is onto. Also, check whether the given function is one-one or not.

$f : R-\{3\} \to R-\{a\}$

$f(x) = \frac{x-2}{x-3}$

Let $f(x) = y = \frac{x-2}{x-3}$

f is onto $\Rightarrow$ Range = Codomain = $R-\{a\}$

Now $xy - 3y = x - 2$

$\Rightarrow x(y-1) = 3y - 2$

$\Rightarrow x = \frac{3y-2}{y-1}$

Since $x \in R-\{3\} \Rightarrow y-1 \neq 0 \Rightarrow y \neq 1$

$\Rightarrow y \in R-\{1\}$

So range of $f = R-\{1\} \subset R-\{a\}$

$\Rightarrow$ $a = 1$ Ans.

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Checking f one-one or not

for $x_1, x_2 \in R - \{3\}$

Let $f(x_1) = f(x_2)$

$$\frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3}$$

$\Rightarrow x_1 = x_2$ (but $x_1 \neq x_2$)

$\Rightarrow f(x_1) = f(x_2)$ only when $x_1 = x_2$

$\Rightarrow$ f is one-one Ans.

34. (a) It is given that function $f(x) = x^4 - 62x^2 + ax + 9$ attains local maximum value at $x = 1$. Find the value of 'a', hence obtain all other points where the given function f(x) attains local maximum or local minimum values.

OR

(b) The perimeter of a rectangular metallic sheet is 300 cm. It is rolled along one of its sides to form a cylinder. Find the dimensions of the rectangular sheet so that volume of cylinder so formed is maximum.

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(a) $f(x) = x^4 - 62x^2 + ax + 9$

local max at $x = 1$, $a = ?$

Points of Local Maximum, Local min.

$f'(1) = 0$ (f has local max. at $x = 1$)

$4x^3 - 124x + a = 0$

$4 - 124 + a = 0$

$a = 120$ Ans

So $f(x) = x^4 - 62x^2 + 120x + 9$

$f'(x) = 4x^3 - 124x + 120 = 0$

$\Rightarrow x^3 - 31x + 30 = 0$

$\Rightarrow (x-1)(x^2 + x - 30) = 0$

$\Rightarrow (x-1)(x+6)(x-5) = 0$

$x = 1$ Point of local maximum (given)

at $x = 5$

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$f''(x) = 4(3x^2 - 31)$

$f''(5) = 4(75 - 31) = +$ $\Rightarrow$ $x = 5$ is point of local minima

at $x = -6$

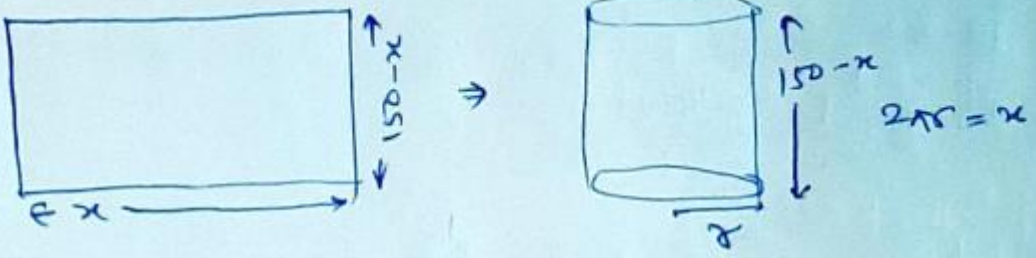
$f''(-6) = 4(3 \cdot 36 - 31) = +$

$\Rightarrow$ $x = -6$ is point of local minimum

So Points of local maximum : $x = 1$

Points of local minimum : $x = 5, \; x = -6$

OR



Perimeter = 300 cm

Let length = x, breadth = $150 - x$

Volume of cyl = $V = \pi r^2 h$

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$= \pi\left(\frac{x}{2\pi}\right)^2 \cdot (150 - x)$

$= \frac{1}{4\pi} x^2 (150 - x) = \frac{1}{4\pi}(150x^2 - x^3)$

for V is maximum $\frac{dV}{dx} = 0$

$\Rightarrow 300x - 3x^2 = 0 \Rightarrow x^2 = 100$

$x = 100$ or ~~0~~

~~So~~ $\frac{d^2V}{dx^2} = \frac{1}{4\pi}(300 - 6x)\Big)_{x=100} = -ve$

$\Rightarrow$ Vol. is max. at $x = 100$

So length = 100 cm, breadth = 50 cm. Ans.

dimensions

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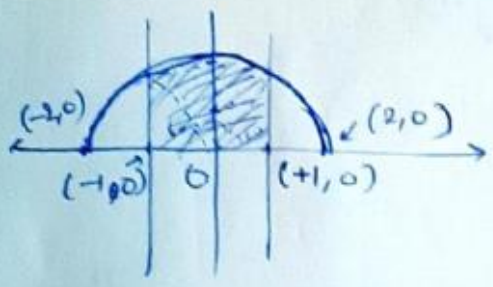
35. Using integration, find the area of the region enclosed between the curve $y = \sqrt{4-x^2}$ and the lines $x = -1$, $x = 1$ and the x-axis.

35

$y = \sqrt{4-x^2}$

$x = -1$, $x = 1$, x axis



$$\text{Area} = \int_{-1}^{1} y \cdot dx$$

$$= \int_{-1}^{1} \sqrt{4-x^2}\, dx$$

$$= 2\int_{0}^{1} \sqrt{4-x^2}\, dx$$

$$= 2\left[\frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^1$$

$$= 2\left[\left(\frac{1}{2}\sqrt{3} + 2\sin^{-1}\frac{1}{2}\right) - (0)\right]$$

$$= \sqrt{3} + 4 \cdot \pi/6 = \frac{2\pi}{3} + \sqrt{3} \text{ Sq. units}$$

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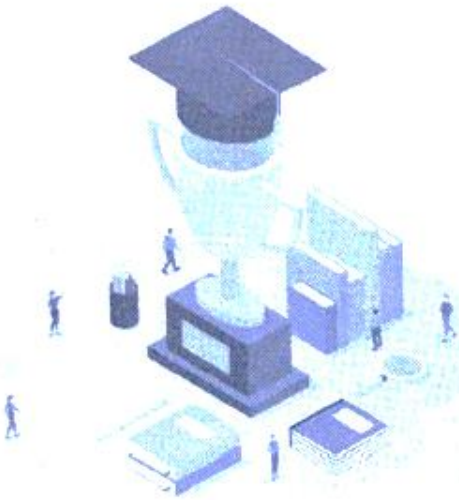
36. A scholarship is a sum of money provided to a student to help him or her pay for education. Some students are granted scholarships based on their academic achievements, while others are rewarded based on their financial needs.



Every year a school offers scholarships to girl children and meritorious achievers based on certain criteria. In the session 2022 – 23, the school offered monthly scholarship of ₹ 3,000 each to some girl students and ₹ 4,000 each to meritorious achievers in academics as well as sports.

In all, 50 students were given the scholarships and monthly expenditure incurred by the school on scholarships was ₹ 1,80,000.

Based on the above information, answer the following questions :

(i) Express the given information algebraically using matrices. *1*

(ii) Check whether the system of matrix equations so obtained is consistent or not. *1*

(iii) (a) Find the number of scholarships of each kind given by the school, using matrices. *2*

OR

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(iii) (b) Had the amount of scholarship given to each girl child and meritorious student been interchanged, what would be the monthly expenditure incurred by the school ? 2

36 Types of scholarships - Girl candidate (Rs. 3000)
Meritorius student (Rs. 4000)

Total no of students = 50

let no. of students (getting Rs. 3000/month) = x
——— (——— 4000/month) = y

ATQ.

(i) $x + y = 50$
$3000x + 4000y = 1,80,000$] $\Rightarrow \begin{pmatrix} 1 & 1 \\ 3000 & 4000 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 50 \\ 1,80,000 \end{pmatrix}$

(ii) in (i) let system is $AX = B$

here $|A| \neq 0 \Rightarrow$ system is consistent

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(iii)

$$X = A^{-1}B$$

$$A^{-1} = \frac{1}{|A|}\,\text{Adj}(A) = \frac{1}{1000}\begin{pmatrix} 4000 & -1 \\ -3000 & 1 \end{pmatrix}$$

$$X = \frac{1}{1000}\begin{pmatrix} 4000 & -1 \\ -3000 & 1 \end{pmatrix}\begin{pmatrix} 50 \\ 180000 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{1000}\begin{pmatrix} 200000 - 180000 \\ -150000 + 180000 \end{pmatrix} = \begin{pmatrix} 20 \\ 30 \end{pmatrix}$$

$x = 20,\ y = 30$

no of students getting Scho. Rs. 3000/mon = 20
4000/mon = 30 Ans

(iii) OR (b) on interchanging

expenses incurred = $30 \times 3000 + 20 \times 4000$
$= 90000 + 80000$
$= 1,70,000$ Ans

12th CBSE MATHS SET – 3 CODE 65/3/3

Case Study – 2

37. Self-study helps students to build confidence in learning. It boosts the self-esteem of the learners. Recent surveys suggested that close to 50% learners were self-taught using internet resources and upskilled themselves.

A student may spend 1 hour to 6 hours in a day in upskilling self. The probability distribution of the number of hours spent by a student is given below :

$$P(X = x) = \begin{cases} kx^2, & \text{for } x = 1, 2, 3 \\ 2kx, & \text{for } x = 4, 5, 6 \\ 0, & \text{otherwise} \end{cases}$$

where x denotes the number of hours.

Based on the above information, answer the following questions :

(i) Express the probability distribution given above in the form of a probability distribution table. 1

(ii) Find the value of k. 1

(iii) (a) Find the mean number of hours spent by the student. 2

OR

(iii) (b) Find $P(1 < X < 6)$. 2

12th CBSE MATHS SET – 3 CODE 65/3/3

37

$$P(X=x) = \begin{cases} kx^2 & x = 1,2,3 \\ 2kx & x = 4,5,6 \\ 0 & \text{otherwise} \end{cases}$$

x = no. of hrs.

(i) Prob. distribution

x	$P(x)$
1	k
2	$4k$
3	$9k$
4	$8k$
5	$10k$
6	$12k$
Σ =	

X = no of hours.
$P(X)$ = Prob (studying X hours)

(ii) $\Sigma P(x) = 1$

$\Rightarrow 43k = 1 \Rightarrow \boxed{k = \frac{1}{43}}$ Ans

(iii) (a) $E(X) = \Sigma x_i P(x_i)$

$= k + 8k + 27k + 32k + 50k + 72k$

$= 190k$

$\boxed{E(X) = \frac{190}{43} = \text{mean}}$ Ans. $\frac{190}{43}$ hrs.

12th CBSE MATHS SET – 3 CODE 65/3/3

(iii) (b) $P(1<X<6) = P(X=2) + P(X=3) + P(X=4) + P(X=5)$

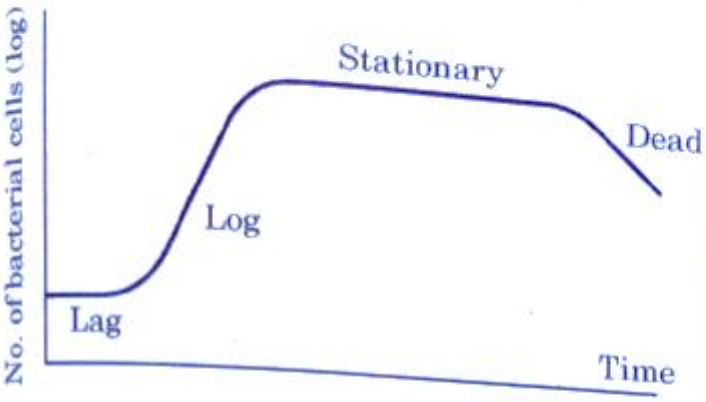
$= 4k + 9k + 8k + 10k$

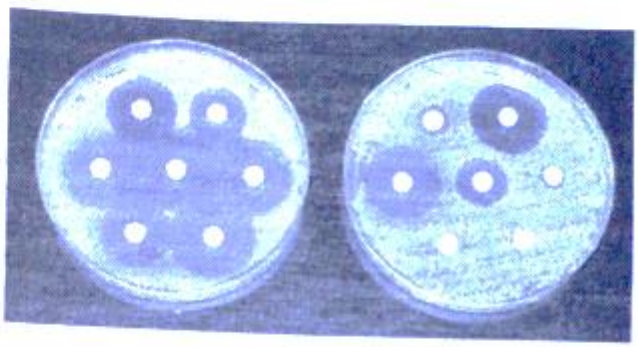
$= 31k$

$= \frac{31}{43}$ Ans.

Case Study – 3

38. A bacteria sample of certain number of bacteria is observed to grow exponentially in a given amount of time. Using exponential growth model, the rate of growth of this sample of bacteria is calculated.





The differential equation representing the growth of bacteria is given as :

$\frac{dP}{dt} = kP$, where P is the population of bacteria at any time 't'.

Based on the above information, answer the following questions :

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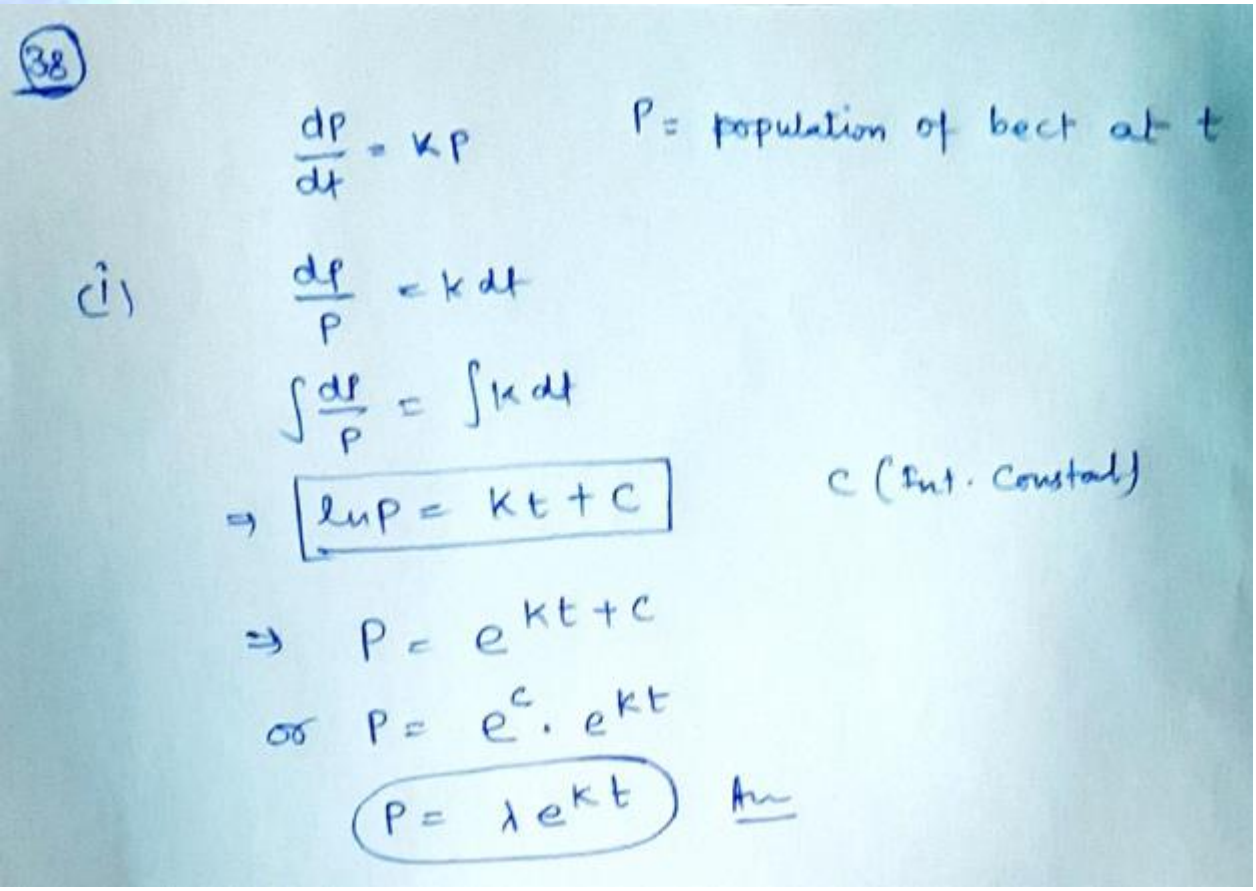
(i) Obtain the general solution of the given differential equation and express it as an exponential function of 't'. 2

(ii) If population of bacteria is 1000 at t = 0, and 2000 at t = 1, find the value of k. 2



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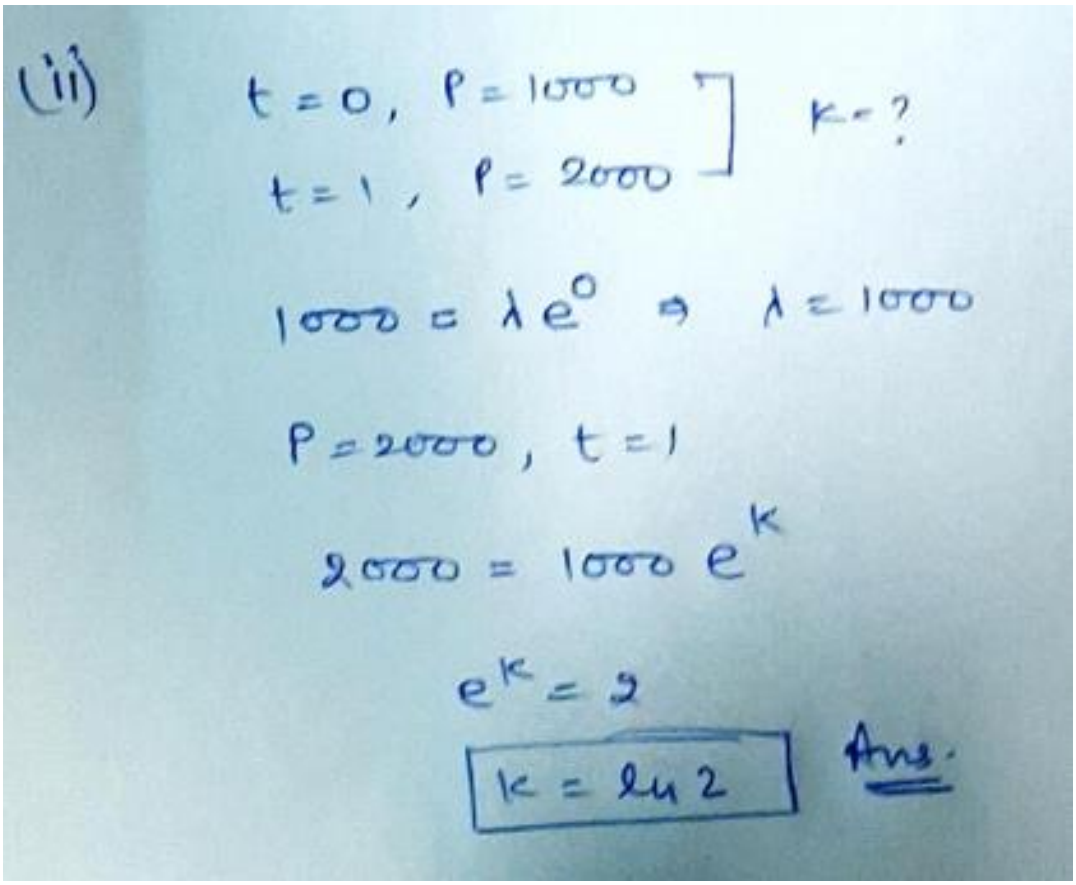

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