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SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. $\int 2^{x+2}dx$ is equal to :

(a) $2^{x+2}+C$ (b) $2^{x+2}\log 2 + C$ formula

(c) $\frac{2^{x+2}}{\log 2}+C$ (d) $2\cdot\frac{2^x}{\log 2}+C$

2. Let A be a skew-symmetric matrix of order 3. If $|A| = x$, then $(2023)^x$ is equal to :

(a) 2023 (b) $\frac{1}{2023}$

(c) $(2023)^2$ (d) 1

$|A| = 0$
$\Rightarrow x = 0$
$\Rightarrow (2023)^0 = 1$

3. $\int_0^2 \sqrt{4-x^2}\,dx$ equals : $= \left[\frac{x}{2}\sqrt{4-x^2}+\frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^2 = 2\sin^{-1}1 = \pi$

D

(a) $2\log 2$ (b) $-2\log 2$

(c) $\frac{\pi}{2}$ (d) π

4. The solution of the differential equation $\frac{dx}{x}+\frac{dy}{y}=0$ is : $\Rightarrow$

(a) $\frac{1}{x}+\frac{1}{y}=C$ (b) $\log x - \log y = C$

(c) $xy = C$ (d) $x + y = C$

$\frac{dx}{x} = -\frac{dy}{y}$ C
$\ln x = -\ln y + c$
$\ln xy = c$
$xy = \lambda$

5. What is the product of the order and degree of the differential equation $\frac{d^2y}{dx^2}\sin y + \left(\frac{dy}{dx}\right)^3\cos y = \sqrt{y}$?

(a) 3 (b) 2

(c) 6 (d) not defined

Order = 2
deg = 1
Product = 2 B

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6. The direction cosines of vector $\overrightarrow{BA}$, where coordinates of A and B are (1, 2, – 1) and (3, 4, 0) respectively, are :

(a) – 2, – 2, – 1 (b) $-\frac{2}{3}, -\frac{2}{3}, -\frac{1}{3}$

(c) 2, 2, 1 (d) $\frac{2}{3}, \frac{2}{3}, \frac{1}{3}$

$\overrightarrow{BA} = -2\hat{i} - 2\hat{j} - \hat{k}$

d r s $= -2, -2, -1$

d c s $= -\frac{2}{3}, -\frac{2}{3}, -\frac{1}{3}$

7. $\vec{a}$ and $\vec{b}$ are two non-zero vectors such that the projection of $\vec{a}$ on $\vec{b}$ is 0. The angle between $\vec{a}$ and $\vec{b}$ is :

(a) $\frac{\pi}{2}$ (b) π

(c) $\frac{\pi}{4}$ (d) 0

$\vec{a}\cdot\vec{b} = 0$

$\Rightarrow a\cos\theta = 0$

$\theta = \pi/2$

8. In Δ ABC, $\overrightarrow{AB} = \hat{i} + \hat{j} + 2\hat{k}$ and $\overrightarrow{AC} = 3\hat{i} - \hat{j} + 4\hat{k}$. If D is mid-point of BC, then vector $\overrightarrow{AD}$ is equal to :

(a) $4\hat{i} + 6\hat{k}$ (b) $2\hat{i} - 2\hat{j} + 2\hat{k}$

(c) $\hat{i} - \hat{j} + \hat{k}$ (d) $2\hat{i} + 3\hat{k}$

$\overrightarrow{AD} = \frac{1}{2}(\overrightarrow{AB} + \overrightarrow{AC})$

$= \frac{1}{2}(4\hat{i} + 0\hat{j} + 6\hat{k})$

D

$= 2\hat{i} + 3\hat{k}$

9. If the point P(a, b, 0) lies on the line $\frac{x+1}{2} = \frac{y+2}{3} = \frac{z+3}{4}$, then (a, b) is :

(a) (1, 2) (b) $\left(\frac{1}{2}, \frac{2}{3}\right)$

(c) $\left(\frac{1}{2}, \frac{1}{4}\right)$ (d) (0, 0)

$\frac{a+1}{2} = \frac{b+2}{3} = \frac{3}{4}$

$a = \frac{1}{2}, b = \frac{1}{4}$

C

10. For any two events A and B, if $P(\bar{A}) = \frac{1}{2}$, $P(\bar{B}) = \frac{2}{3}$ and $P(A \cap B) = \frac{1}{4}$, then $P\left(\frac{\bar{A}}{\bar{B}}\right)$ equals :

wrong no option

Bonus, no option available

(a) $\frac{3}{8}$ Correct ans $= \frac{5}{8}$ (b) $\frac{8}{9}$

(c) $\frac{1}{8}$ (d) $\frac{1}{4}$

$P(A) = \frac{1}{2}$ $P(B) = \frac{1}{3}$

$P(A\cap B) = \frac{1}{4} \Rightarrow P(A\cup B) = \frac{7}{12}$

$P(\bar{A}/\bar{B}) = \frac{P(\bar{A}\cap\bar{B})}{P(\bar{B})}$

$= \frac{P(\overline{A\cup B})}{P(\bar{B})} = \frac{5/12}{2/3}$

$= 5/8$

11. The value of k for which function $f(x) = \begin{cases} x^2, & x \geq 0 \\ kx, & x < 0 \end{cases}$ is differentiable at x = 0 is :

(a) 1 (b) 2

(c) any real number (d) 0

$f'(0^-) = f'(0^+)$

$k = 2\cdot 0$

$k = 0$

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12. If $y = \dfrac{\cos x - \sin x}{\cos x + \sin x}$, then $\dfrac{dy}{dx}$ is :

(a) $-\sec^2\left(\dfrac{\pi}{4} - x\right)$ (b) $\sec^2\left(\dfrac{\pi}{4} - x\right)$

(c) $\log\left|\sec\left(\dfrac{\pi}{4} - x\right)\right|$ (d) $-\log\left|\sec\left(\dfrac{\pi}{4} - x\right)\right|$

$$\boxed{y = \tan\left(\frac{\pi}{4} - x\right) \qquad \frac{dy}{dx} = -\sec^2\left(\frac{\pi}{4} - x\right)}$$

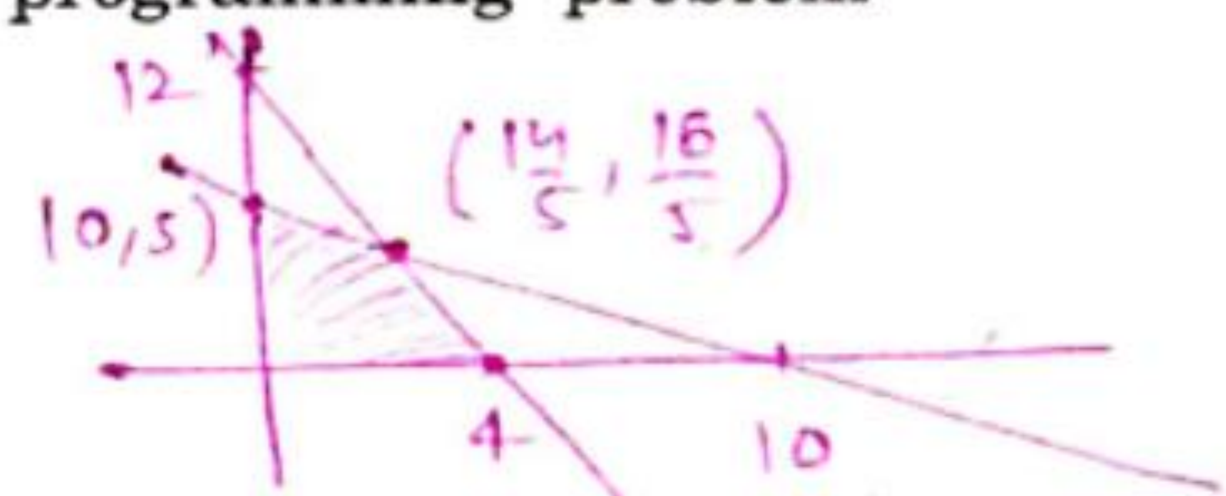
13. The number of feasible solutions of the linear programming problem given as

Maximize $z = 15x + 30y$ subject to constraints :

$3x + y \leq 12,\ x + 2y \leq 10,\ x \geq 0,\ y \geq 0$ is

(a) 1 (b) 2

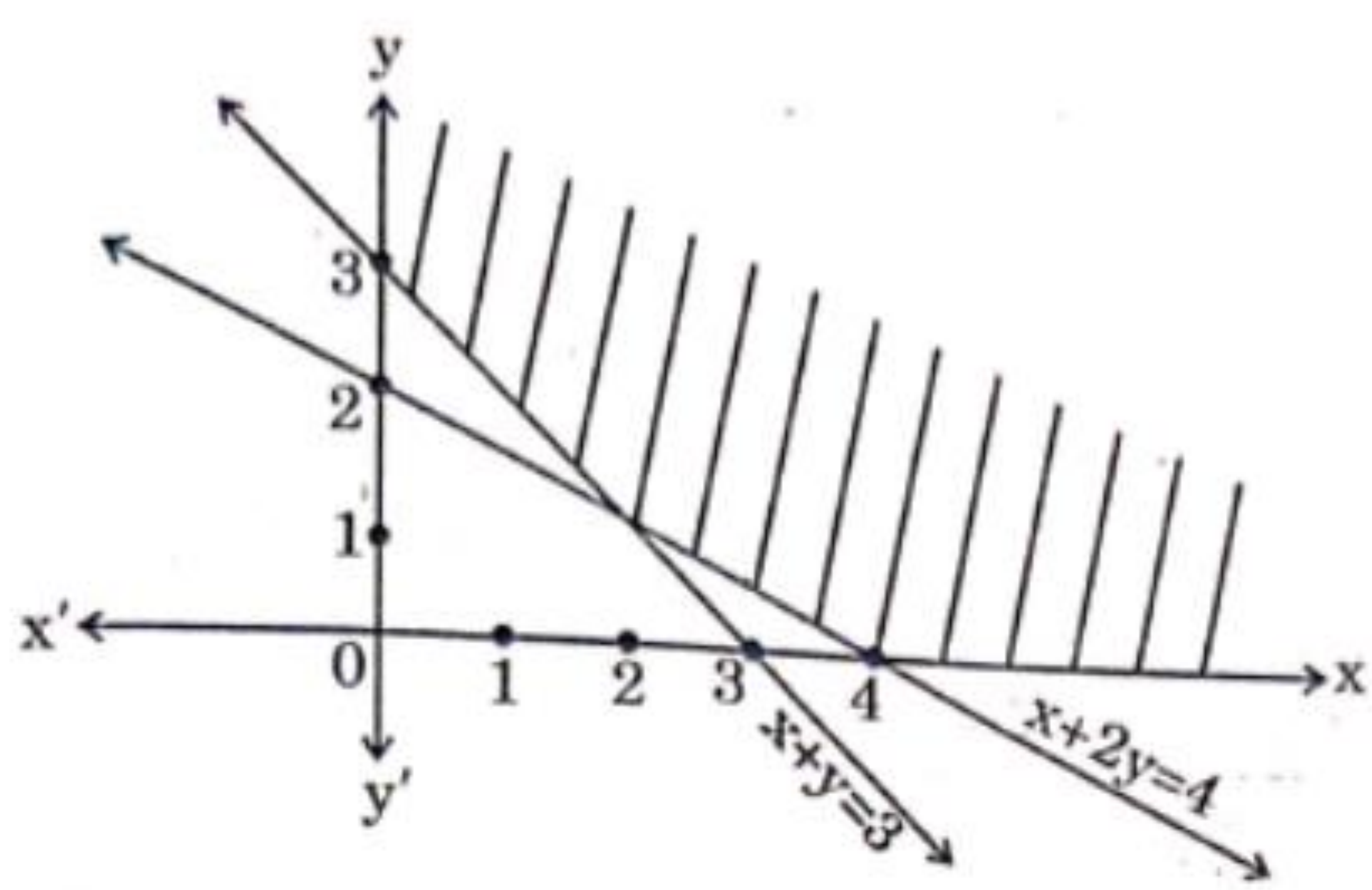
(c) 3 (d) infinite



at $(0,5)$ & $\left(\frac{14}{5}, \frac{16}{5}\right)$ $z_{max} = 150$

So there are two points for maximum value of z in feasible region.

14. The feasible region of a linear programming problem is shown in the figure below :



$x + 2y \geq 4$
$x + y \geq 3$
$x, y \geq 0$

Which of the following are the possible constraints ?

(a) $x + 2y \geq 4,\ x + y \leq 3,\ x \geq 0,\ y \geq 0$

(b) $x + 2y \leq 4,\ x + y \leq 3,\ x \geq 0,\ y \geq 0$

(c) $x + 2y \geq 4,\ x + y \geq 3,\ x \geq 0,\ y \geq 0$

(d) $x + 2y \geq 4,\ x + y \geq 3,\ x \leq 0,\ y \leq 0$

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15. If $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$, then $B'A'$ is equal to :

(a) $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$$B'A' = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$B'A' = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} \quad (B)$$

16. If $A . (\text{adj } A) = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, then the value of $|A| + |\text{adj } A|$ is equal to :

(a) 12   (b) 9

(c) 3   (d) 27

$A \cdot Adj(A) = |A| I$

$\Rightarrow |A| = 3$

$|Adj A| = |A|^2 = 9$

$|A| + |Adj A| = 12$

17. A and B are skew-symmetric matrices of same order. AB is symmetric, if :

(a) $AB = O$ (b) $AB = -BA$

(c) $AB = BA$ (d) $BA = O$

Given $A^T = -A$, $B^T = -B$

we want $(AB)^T = AB$

$\Rightarrow B^T \cdot A^T = AB$

$\Rightarrow (-B)(-A) = AB$

$\Rightarrow BA = AB$ (condition)

18. For what value of $x \in \left[0, \frac{\pi}{2}\right]$, is $A + A' = \sqrt{3}\, I$, where

$A = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$?

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$

(c) 0   (d) $\frac{\pi}{2}$

$$A + A' = \begin{pmatrix} 2\cos x & 0 \\ 0 & 2\cos x \end{pmatrix} = \begin{pmatrix} \sqrt{3} & 0 \\ 0 & \sqrt{3} \end{pmatrix}$$

$\Rightarrow \cos x = \sqrt{3}/2$

$x = \pi/6$ (B)

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19. *Assertion (A) :* A line through the points (4, 7, 8) and (2, 3, 4) is parallel to a line through the points (– 1, – 2, 1) and (1, 2, 5). T

Reason (R): Lines $\vec{r} = \vec{a_1} + \lambda \vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu \vec{b_2}$ are parallel if $\vec{b_1} \cdot \vec{b_2} = 0$. F

(C)

20. *Assertion (A) :* Range of $[\sin^{-1} x + 2\cos^{-1} x]$ is $[0, \pi]$. <u>False</u> as range $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$

Reason (R) : Principal value branch of $\sin^{-1} x$ has range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. T

(D)

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This section comprises very short answer (VSA) type questions of 2 marks each.

21. Consider the statement "There exists at least one value of $b \in \mathbb{R}$ for which $f(x) = \frac{b}{x}$, $b \neq 0$ is strictly increasing in $\mathbb{R} - \{0\}$."

State True or False. Justify. True if $b<0$ →

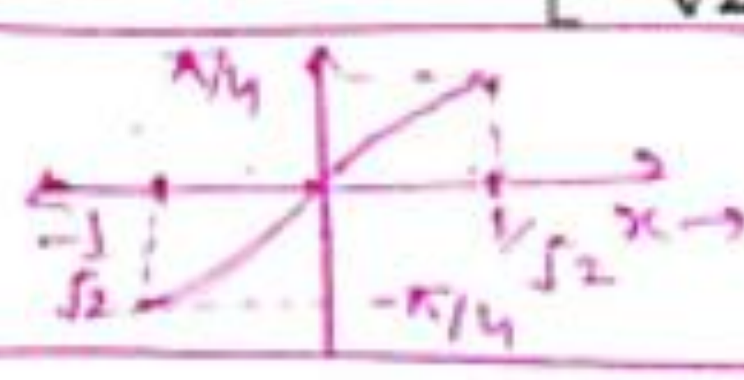
$f'(x) > 0$ (for Incr)
$-\frac{b}{x^2} > 0$
$-b > 0$ $(x^2 > 0)$
$b < 0$
$\forall\, b \in \mathbb{R}^-$, f is ↑.
TRUE

22. (a) Evaluate : $3\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) + 2\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) + \cos^{-1}(0)$

$\frac{3\pi}{4} + 2 \cdot \frac{\pi}{6} + \frac{\pi}{2} = \frac{19\pi}{12}$ Ans.

OR

(b) Draw the graph of $f(x) = \sin^{-1} x$, $x \in \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$. Also, write range of f(x).

Graph 

, range = $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

23. (a) If $y = x^{\frac{1}{x}}$, then find $\frac{dy}{dx}$ at $x = 1$. ⇒ Ans = 1

OR

(b) If $x = a\sin 2t$, $y = a(\cos 2t + \log\tan t)$, then find $\frac{dy}{dx}$. $= \cot 2t$

24. If $\vec{r} = 3\hat{i} - 2\hat{j} + 6\hat{k}$, find the value of $(\vec{r} \times \hat{j}) \cdot (\vec{r} \times \hat{k}) - 12$. Ans = 0

25. Find the value of p, so that lines $\frac{x-1}{-2} = \frac{y-4}{3p} = \frac{z-3}{4}$ and $\frac{x-2}{4p} = \frac{y-5}{2} = \frac{1-z}{7}$ are perpendicular to each other.

drs $(l_1) = -2, 3p, 4$
drs $(l_2) = 4p, 2, -7$
Now $-8p + 6p - 28 = 0$
⇒ $p = -14$ Ans.

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This section comprises short answer (SA) type questions of 3 marks each.

26. Find :

$$\int \frac{e^x}{\sqrt{e^{2x} - 4e^x - 5}}\,dx$$

$e^x = t, \Rightarrow e^x dx = dt$

$$I = \int \frac{dt}{\sqrt{t^2-4t-5}} = \int \frac{dt}{\sqrt{(t-2)^2-9}}$$

$$= \log\left[(t-2)+\sqrt{t^2-4t-5}\right] = \log\left(e^x-2+\sqrt{e^{2x}-4e^x-5}\right) + C$$

Ans.

27. (a) Find :

$$\int \frac{\cos x}{\sin 3x}\,dx = \int \frac{dt}{3t-4t^3} = \int \frac{1}{3}\left(\frac{1}{t}+\frac{4t}{3-4t^2}\right)$$

$t = \sin x,\ \cos x\,dx = dt$

$$= \frac{1}{3}\ln t - \frac{1}{6}\int \frac{-8t}{3-4t^2}\,dt$$

$$= \frac{1}{3}\ln t - \frac{1}{6}\ln(3-4t^2) + C$$

$$= \frac{1}{3}\ln \sin x - \frac{1}{6}\ln(3-4\sin^2 x) + C \quad \text{Ans}$$

OR

(b) Find :

$$\int x^2 \log(x^2+1)\,dx$$

by parts $I = \frac{x^3}{3}\ln(1+x^2) + \frac{2}{3}x - \frac{2}{9}x^3 - \frac{2}{3}\tan^{-1}x + C$ Ans.

27. (b) $I = \int x^2 \ln(1+x^2)\,dx$ (using by parts)

$$I = \ln(1+x^2)\int x^2 dx - \int \frac{2x}{1+x^2}\cdot\frac{x^3}{3}\,dx$$

$$= \frac{x^3}{3}\ln(1+x^2) - \frac{2}{3}\int \frac{x^4}{1+x^2}\,dx$$

manipulating

$$= \frac{x^3}{3}\ln(1+x^2) + \frac{2}{3}\int \frac{-x^4+1-1}{1+x^2}\,dx$$

$$= \frac{x^3}{3}\ln(1+x^2) + \frac{2}{3}\int \frac{1-x^4}{1+x^2}\,dx - \frac{2}{3}\int \frac{1}{1+x^2}\,dx$$

$$= \frac{x^3}{3}\ln(1+x^2) + \frac{2}{3}\int (1-x^2)\,dx - \frac{2}{3}\tan^{-1}x$$

$$\boxed{I = \frac{x^3}{3}\ln(1+x^2) + \frac{2}{3}x - \frac{2}{9}x^3 - \frac{2}{3}\tan^{-1}x + C}$$ Ans.

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28. Solve the following linear programming problem graphically :

Maximize $z = 3x + 9y$

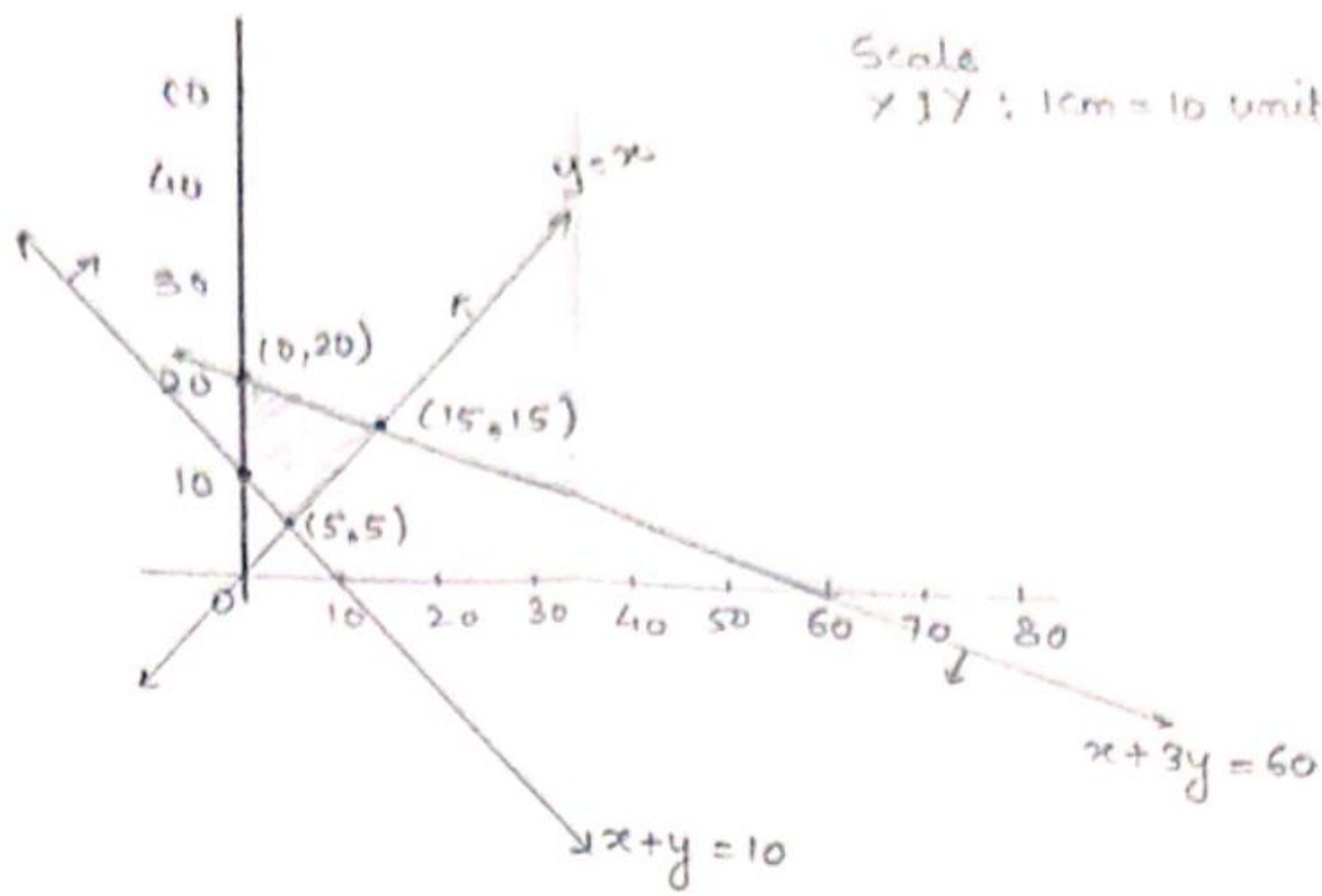
subject to the constraints

$$x + y \geq 10,$$
$$x + 3y \leq 60,$$
$$x \leq y,$$
$$x \geq 0, y \geq 0.$$

z is max at $(15,15)$ or $(0,20)$

$Z_{max} = 180$

28



$Z = 3x + 9y$ at $(5,5)$ $Z = 60$

at $(15,15)$, $Z = 180$ } maximum

at $(0,20)$ $Z = 180$

at $(0,10)$ $Z = 90$

So $Z_{max} = 180$ at $(15,15)$ or $(0,20)$

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29. (a) A pair of dice is thrown simultaneously. If X denotes the absolute difference of numbers obtained on the pair of dice, then find the probability distribution of X.

X	0	1	2	3	4	5
P(X)	6/36	10/36	8/36	6/36	4/36	2/36

Ans-

OR

(b) There are two coins. One of them is a biased coin such that P (head) : P (tail) is 1 : 3 and the other coin is a fair coin. A coin is selected at random and tossed once. If the coin showed head, then find the probability that it is a biased coin. Ans $= \frac{1}{3}$

Biased coin
$P(H) = 1/4$
$P(T) = 3/4$

$$P\left(\frac{\text{biased coin}}{\text{coin shows H}}\right) = \frac{P(B)\cdot P(H)}{P(B)P(H)+P(N)P(H)} = \frac{\frac{1}{2}\cdot\frac{1}{4}}{\frac{1}{2}\cdot\frac{1}{4}+\frac{1}{2}\cdot\frac{1}{2}} = \frac{1}{3}$$

$P(B) = P($biased coin is chosen$)$, $P(N) = P($normal is chosen$)$

30. (a) Find the general solution of the differential equation :

$$\frac{d}{dx}(xy^2) = 2y(1+x^2)$$

$$2xy\frac{dy}{dx} + y^2 = 2y + 2x^2y \Rightarrow \frac{dy}{dx} + \frac{y}{2x} = \frac{1}{x} + x$$

$$IF = e^{\int \frac{1}{2x}dx} = e^{\frac{1}{2}\ln x} = \sqrt{x}$$

Sol^n: $y(\sqrt{x}) = \int \sqrt{x}\left(x+\frac{1}{x}\right)dx$

$$\Rightarrow \boxed{y\sqrt{x} = 2\sqrt{x} + \frac{2}{5}x^2\sqrt{x} + C}$$ Ans.

OR

(b) Solve the following differential equation :

$$xe^{\frac{y}{x}} - y + x\frac{dy}{dx} = 0$$

$y/x = t \Rightarrow y = tx \Rightarrow \frac{dy}{dx} = t + x\frac{dt}{dx}$

DE $\Rightarrow e^t - \cancel{t} + \cancel{t} + x\frac{dt}{dx} = 0$

$\Rightarrow \frac{dt}{e^t} = -\frac{dx}{x} \Rightarrow e^{-t} = \ln x + C$

$\boxed{e^{-y/x} = \ln x + C}$ Ans.

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$$\Rightarrow \frac{dt}{e^t} = -\frac{dx}{x} \Rightarrow e^{-t} = \ln x + c$$

$$\boxed{e^{-y/x} = \ln x + C} \quad \text{Ans.}$$

31. Evaluate :

$$\int_{-\pi/2}^{\pi/2} \frac{\sin^{100} x}{\sin^{100} x + \cos^{100} x}\, dx = \frac{\pi}{2} \quad \text{Ans.}$$

(31) $I = \int_{-\pi/2}^{\pi/2} \frac{\sin^{100} x}{\sin^{100} x + \cos^{100} x}\, dx$

$$I = 2\int_0^{\pi/2} \frac{\sin^{100} x}{\sin^{100} x + \cos^{100} x}\, dx \quad (f(-x) = f(x)) \quad \text{———} \ (1)$$

$$\Rightarrow I = 2\int_0^{\pi/2} \frac{\cos^{100} x}{\sin^{100} x + \cos^{100} x}\, dx \quad \text{———} \ (2) \quad \left(\text{using } \int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx\right)$$

(1) + (2)

$$\cancel{2}I = \cancel{2}\int_0^{\pi/2} 1 \cdot dx \Rightarrow \boxed{I = \pi/2} \quad \text{Ans.}$$

32. (a) If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$, then show that $A^3 - 6A^2 + 7A + 2I = O$.

OR

(b) If $A = \begin{bmatrix} 3 & 2 \\ 5 & -7 \end{bmatrix}$, then find A^{-1} and use it to solve the following system of equations :

$3x + 5y = 11, \ 2x - 7y = -3.$

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32. [a] $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}$ Show that $A^3 - 6A^2 + 7A + 2I = 0$

$$A^2 = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}\begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix} = \begin{pmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{pmatrix}$$

$$A^3 = \begin{pmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{pmatrix}\begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix} = \begin{pmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{pmatrix}$$

$$A^3 - 6A^2 + 7A + 2I = \begin{pmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{pmatrix} - \begin{pmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{pmatrix} + \begin{pmatrix} 7 & 0 & 14 \\ 0 & 14 & 7 \\ 14 & 0 & 21 \end{pmatrix} + \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \text{Hence Proved}$$

32. (b) $A = \begin{pmatrix} 3 & 2 \\ 5 & -7 \end{pmatrix}$

Finding A^{-1}

$$C_A = \begin{pmatrix} -7 & -5 \\ -2 & 3 \end{pmatrix}$$

$$Adj(A) = \begin{pmatrix} -7 & -2 \\ -5 & 3 \end{pmatrix}$$

$$A^{-1} = \frac{1}{|A|} \cdot Adj(A) \quad \text{Ans.}$$

$$\boxed{A^{-1} = \frac{1}{-31}\begin{pmatrix} -7 & -2 \\ -5 & 3 \end{pmatrix}}$$

Since $A^{-1} = \frac{-1}{31}\begin{pmatrix} -7 & -2 \\ -5 & 3 \end{pmatrix}$

$$\boxed{(A^T)^{-1} = \frac{-1}{31}\begin{pmatrix} -7 & -5 \\ -2 & 3 \end{pmatrix}}$$

$$\Rightarrow \begin{pmatrix} 3 & +5 \\ 2 & -7 \end{pmatrix}^{-1} = \frac{-1}{31}\begin{pmatrix} -7 & -5 \\ -2 & 3 \end{pmatrix}$$

Solve $3x + 5y = 11$
$2x - 7y = -3$

$$\Rightarrow \begin{pmatrix} 3 & 5 \\ 2 & -7 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 11 \\ -3 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 & 5 \\ 2 & -7 \end{pmatrix}^{-1}\begin{pmatrix} 11 \\ -3 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{-1}{31}\begin{pmatrix} -7 & -5 \\ -2 & 3 \end{pmatrix}\begin{pmatrix} 11 \\ -3 \end{pmatrix}$$

$$= \frac{-1}{31}\begin{pmatrix} -62 \\ -31 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$\Rightarrow \boxed{x = 2,\ y = 1} \quad \text{Ans.}$$

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33. (a) Find the value of b so that the lines $\frac{x-1}{2}=\frac{y-b}{3}=\frac{z-3}{4}$ and $\frac{x-4}{5}=\frac{y-1}{2}=z$ are intersecting lines. Also, find the point of intersection of these given lines.

OR

(b) Find the equations of all the sides of the parallelogram ABCD whose vertices are A(4, 7, 8), B(2, 3, 4), C(– 1, – 2, 1) and D(1, 2, 5). Also, find the coordinates of the foot of the perpendicular from A to CD.

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33. (a) $L_1 : \frac{x-1}{2}=\frac{y-b}{3}=\frac{z-3}{4}=\lambda$ $\quad$ $L_2 : \frac{x-4}{5}=\frac{y-1}{2}=\frac{z-0}{1}=\mu$

General point $(2\lambda+1, 3\lambda+b, 4\lambda+3)$ $\quad$ Gen pt : $(5\mu+4, 2\mu+1, \mu)$

Lines are insecting $\Rightarrow (2\lambda+1, 3\lambda+b, 4\lambda+3) = (5\mu+4, 2\mu+1, \mu)$

$\Rightarrow 2\lambda+1=5\mu+4 \Rightarrow 2\lambda-5\mu=3$ —— (1)

$3\lambda+b=2\mu+1 \Rightarrow 3\lambda-2\mu=1-b$ —— (2)

$4\lambda+3=\mu \Rightarrow 4\lambda-\mu=-3$ —— (3)

Solving (1) & (3) $\Rightarrow$

$$\begin{array}{r} 2\lambda-5\mu=3 \\ -\;\; 20\lambda-5\mu=-15 \\ \hline -18\lambda \;\;=18 \end{array} \Rightarrow \boxed{\lambda=-1,\ \mu=-1}$$

from (2) $\Rightarrow -3+2=1-b \Rightarrow \boxed{b=2}$ Ans.

Point of intersection $(-1,-1,-1)=(-1,-1,-1)$

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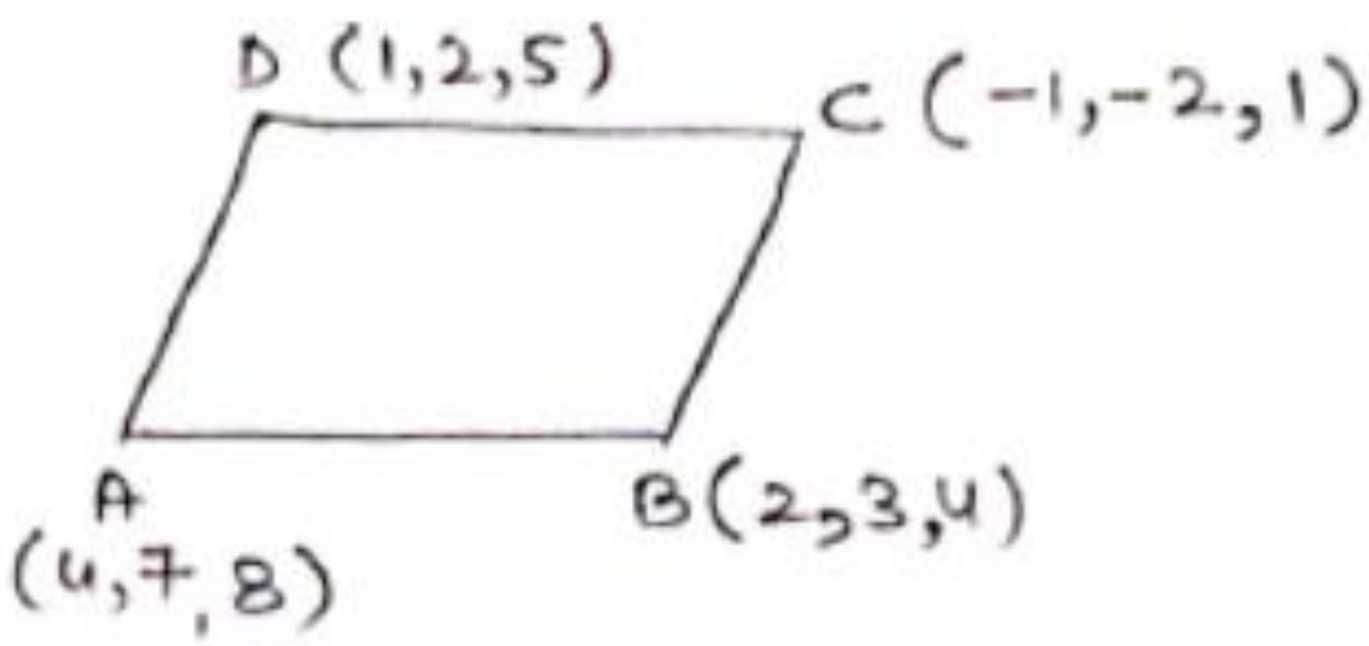
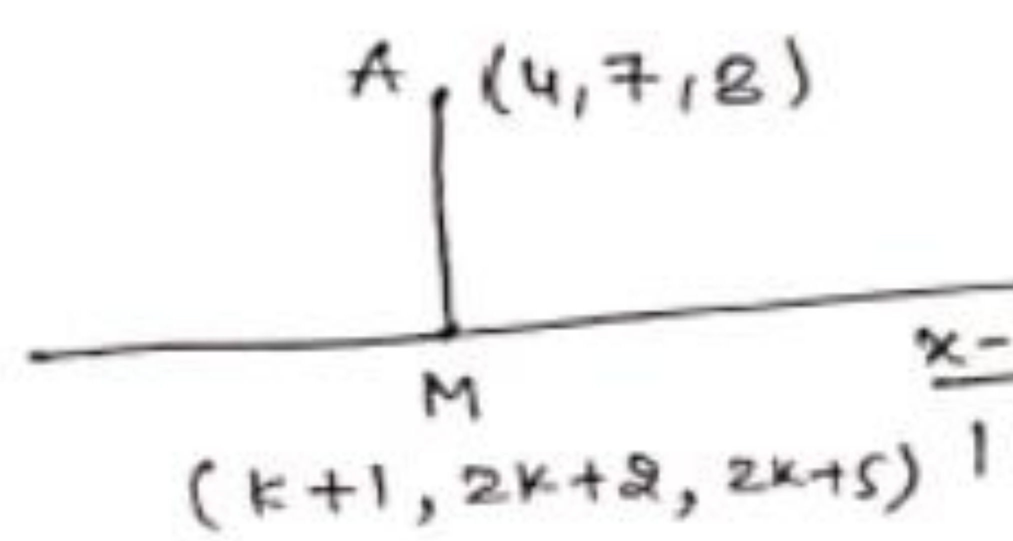
33. (b) 4



	drs	Point	Eqn	
Eqn of AB :	2,4,4 ≃ 1,2,2	(2,3,4)	$\frac{x-2}{1}=\frac{y-3}{2}=\frac{z-4}{2}$	Ans.
BC	3,5,3	(2,3,4)	$\frac{x-2}{3}=\frac{y-3}{5}=\frac{z-4}{3}$	Ans.
Eqn of CD	2,4,4 ≃ 1,2,2	(1,2,5)	$\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-5}{2}$	Ans.
Eqn of AD	3,5,3	(1,2,5)	$\frac{x-1}{3}=\frac{y-2}{5}=\frac{z-5}{3}$	Ans.

Foot of ⊥ from A to CD = M



$\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-5}{2}=k$

$M = (k+1,\ 2k+2,\ 2k+5)$

drs of AM $= k-3,\ 2k-5,\ 2k-3$

$AM \perp CD \Rightarrow (k-3)+2(2k-5)+2(2k-3)=0$

$\Rightarrow 9k-19=0 \Rightarrow k=\frac{19}{9}$

So $M=\left(\frac{19}{9}+1,\ \frac{38}{9}+2,\ \frac{38}{9}+5\right)=\left(\frac{28}{9},\ \frac{56}{9},\ \frac{83}{9}\right)$ Ans.

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34. Prove that a function $f : [0, \infty) \to [-5, \infty)$ defined as $f(x) = 4x^2 + 4x - 5$ is both one-one and onto.

34 $f : [0, \infty) \to [-5, \infty)$ $f(x) = 4x^2 + 4x - 5$

Check for one-one

Let for $x_1, x_2 \in [-5, \infty)$

$f(x_1) = f(x_2)$

$4x_1^2 + 4x_1 - 5 = 4x_2^2 + 4x_2 - 5$

$4(x_1^2 - x_2^2) + 4(x_1 - x_2) = 0$

$\Rightarrow (x_1 - x_2)(x_1 + x_2 + 1) = 0$

$\Rightarrow x_1 + x_2 = -1 \quad (x_1 \neq x_2)$

for $x_1, x_2 \in (0, \infty)$, $x_1 + x_2$ can't be -1.

So no two $x_1, x_2 \in [0, 1)$ are possible so f is one-one

Check for onto

Let $y = f(x)$

$y = 4x^2 + 4x - 5$

$\Rightarrow y = (2x+1)^2 - 6$

$\forall$ $x \in [0, \infty)$ means for $x \geq 0$

$2x + 1 \geq 1$

$\Rightarrow (2x+1)^2 \geq 1$

$\Rightarrow (2x+1)^2 - 6 \geq -5$

$\Rightarrow y \geq -5$

$\Rightarrow y \in [-5, \infty)$

So range = codomain

function is onto

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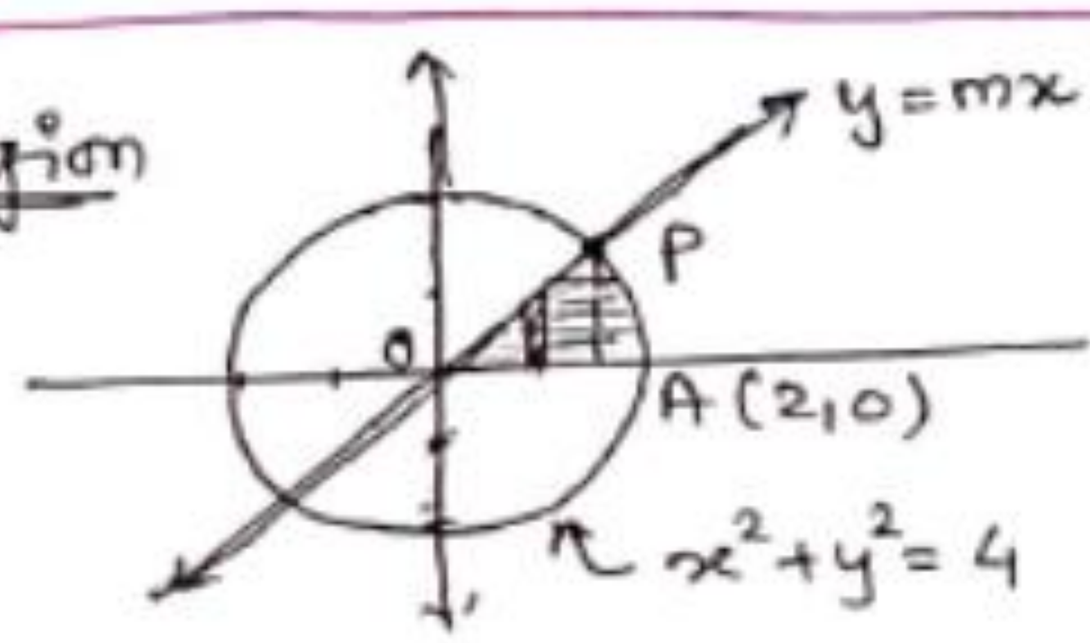
35. The area of the region bounded by the line $y = mx$ $(m > 0)$, the curve $x^2 + y^2 = 4$ and the x-axis in the first quadrant is $\frac{\pi}{2}$ units. Using integration, find the value of m. $m = 1$ Ans.

35. 1. Region



2. Point of intersection : P

$$x^2 + m^2x^2 = 4$$

$$x = \frac{2}{\sqrt{1+m^2}} = P$$

3. Area of Shaded reg $= \frac{\pi}{2}$

$$\int_0^{2/\sqrt{1+m^2}} y\,dx + \int_{\frac{2}{\sqrt{1+m^2}}}^{2} y\,dx = \pi/2$$

$$\Rightarrow \int_0^P mx\,dx + \int_P^2 \sqrt{4-x^2}\,dx = \pi/2$$

$$\Rightarrow \frac{m}{2}\left[x^2\right]_0^{\frac{2}{\sqrt{1+m^2}}} + \left[\frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_{\frac{2}{\sqrt{1+m^2}}}^{2} = \pi/2$$

$$\frac{m}{2}\left[\frac{4}{1+m^2} - 0\right] + \left[\frac{4}{2}\sin^{-1}(1)\right] - \left[\frac{1}{\sqrt{1+m^2}}\sqrt{4 - \frac{4}{1+m^2}} + \frac{4}{2}\sin^{-1}\frac{1}{\sqrt{1+m^2}}\right] = \frac{\pi}{2}$$

$$\Rightarrow \frac{2m}{1+m^2} + \pi - \frac{2m}{1+m^2} - 2\sin^{-1}\frac{1}{\sqrt{1+m^2}} = \frac{\pi}{2}$$

$$\Rightarrow 2\sin^{-1}\frac{1}{\sqrt{1+m^2}} = \frac{\pi}{2} \Rightarrow \sin^{-1}\frac{1}{\sqrt{1+m^2}} = \frac{\pi}{4}$$

$$1 + m^2 = 2$$

$$\boxed{m = 1}$$ Ans.

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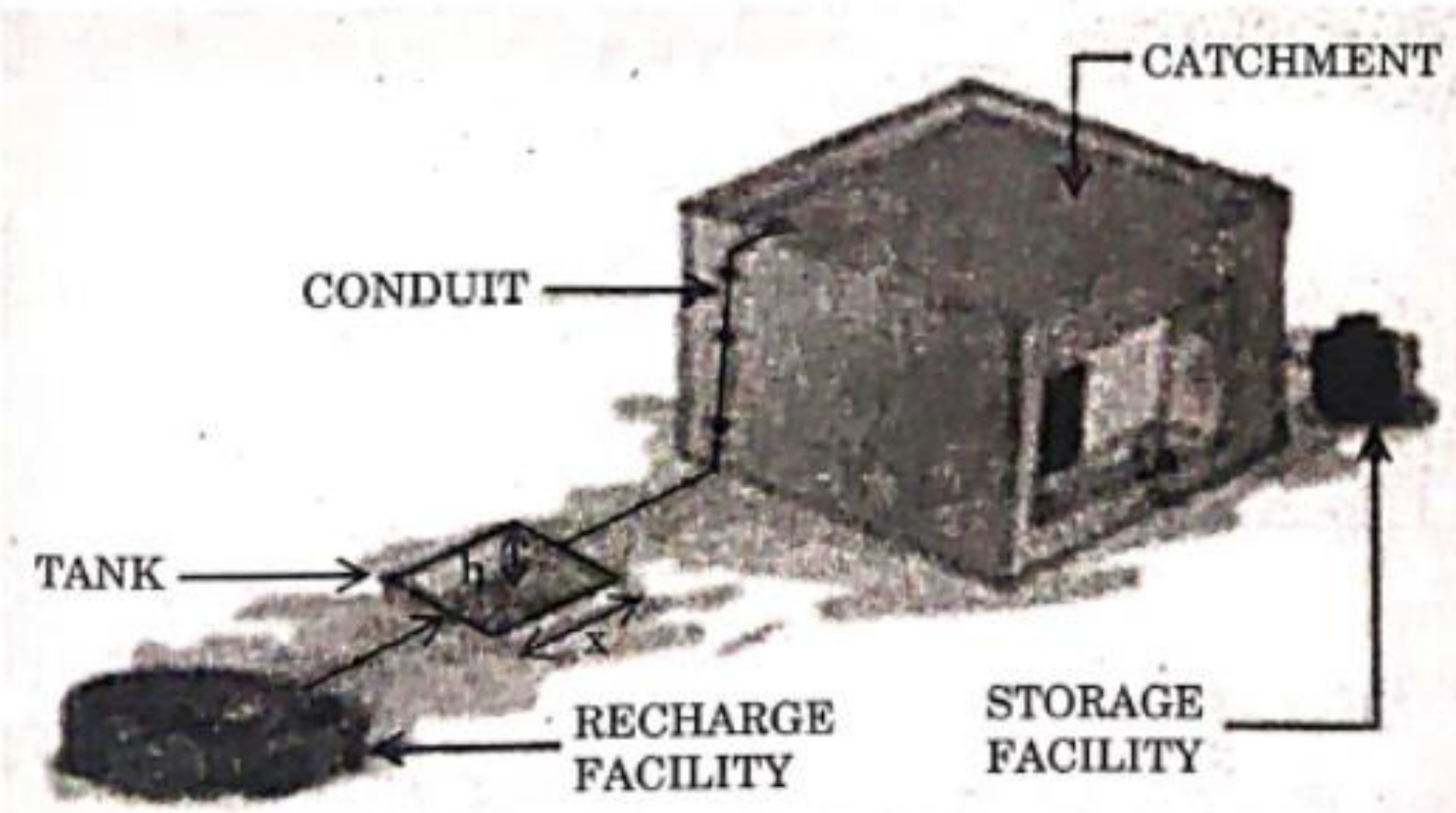
36. In order to set up a rain water harvesting system, a tank to collect rain water is to be dug. The tank should have a square base and a capacity of 250 m^3. The cost of land is ₹ 5,000 per square metre and cost of digging increases with depth and for the whole tank, it is ₹ 40,000 h^2, where h is the depth of the tank in metres. x is the side of the square base of the tank in metres.

ELEMENTS OF A TYPICAL RAIN WATER HARVESTING SYSTEM



Based on the above information, answer the following questions :

(i) Find the total cost C of digging the tank in terms of x. $C = 5000\left(x^2 + \frac{500,000}{x^4}\right)$ 1

(ii) Find $\frac{dC}{dx}$. $= 10^4\left(x - \frac{10^6}{x^3}\right)$ 1

(iii) (a) Find the value of x for which cost C is minimum. 2

$x = 10\sqrt{10}$ m

OR

(iii) (b) Check whether the cost function C(x) expressed in terms of x is increasing or not, where x > 0. 2

f is ~~no~~ increasing for $x > 10\sqrt{10}$
~~and~~ not for $x > 0$

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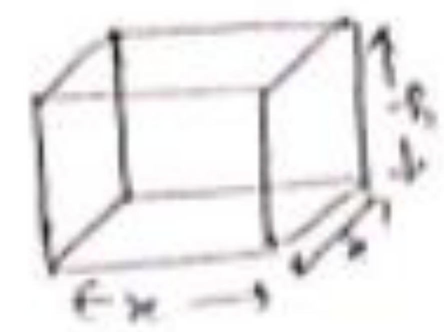
36 : Tank



Cost of land = Rs. 5000/m²
cost of digging = 40,000 h^2 (h = height)
vol.

(i) $C = 5000x^2 + 40000h^2$

since $250 = x^2 h \Rightarrow h = \frac{250}{x^2}$

$C = 5000x^2 + 40000 \cdot \left(\frac{250}{x^2}\right)^2$

$C = 5000x^2 + \frac{40{,}000 \times 62500}{x^4} = 5000x^2 + \frac{2500000000}{x^4}$

$= \boxed{5000\left(x^2 + \frac{500000}{x^4}\right)}$ Ans.

~~(ii) $\frac{dC}{dx} = 10000x - 5 \ldots x$~~

(ii) $\frac{dC}{dx} = \boxed{5000\left(2x - \frac{4 \times 500000}{x^3}\right)}$ Ans.

(iii) a for min value of C, $\frac{dC}{dx} = 0$

$\Rightarrow x^4 = 1000000 = 10^6$

$x^2 = 10^3 = 1000$

since x = only one positive value of $\frac{dC}{dx}$ so C will be min for it, according to the question.

$x = \sqrt{1000} \Rightarrow \boxed{x = 10\sqrt{10}\ m}$ Ans.

(iii) b check C(x) is increasing for x > 0 or not

$\frac{dC}{dx} > 0$

$5000\left(2x - \frac{2000000}{x^3}\right) > 0$

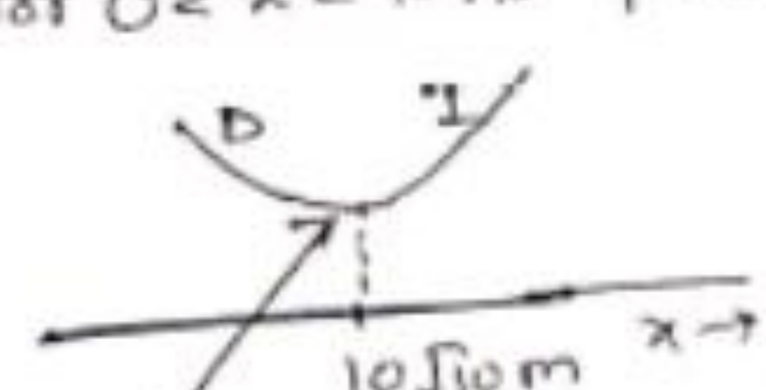
$\Rightarrow 10^4\left(x - \frac{10^6}{x^3}\right) > 0$

$\Rightarrow x^4 > 10^6 \quad (x > 0)$

$\Rightarrow (x^2 - 10^3)(x^2 + 10^3) > 0$

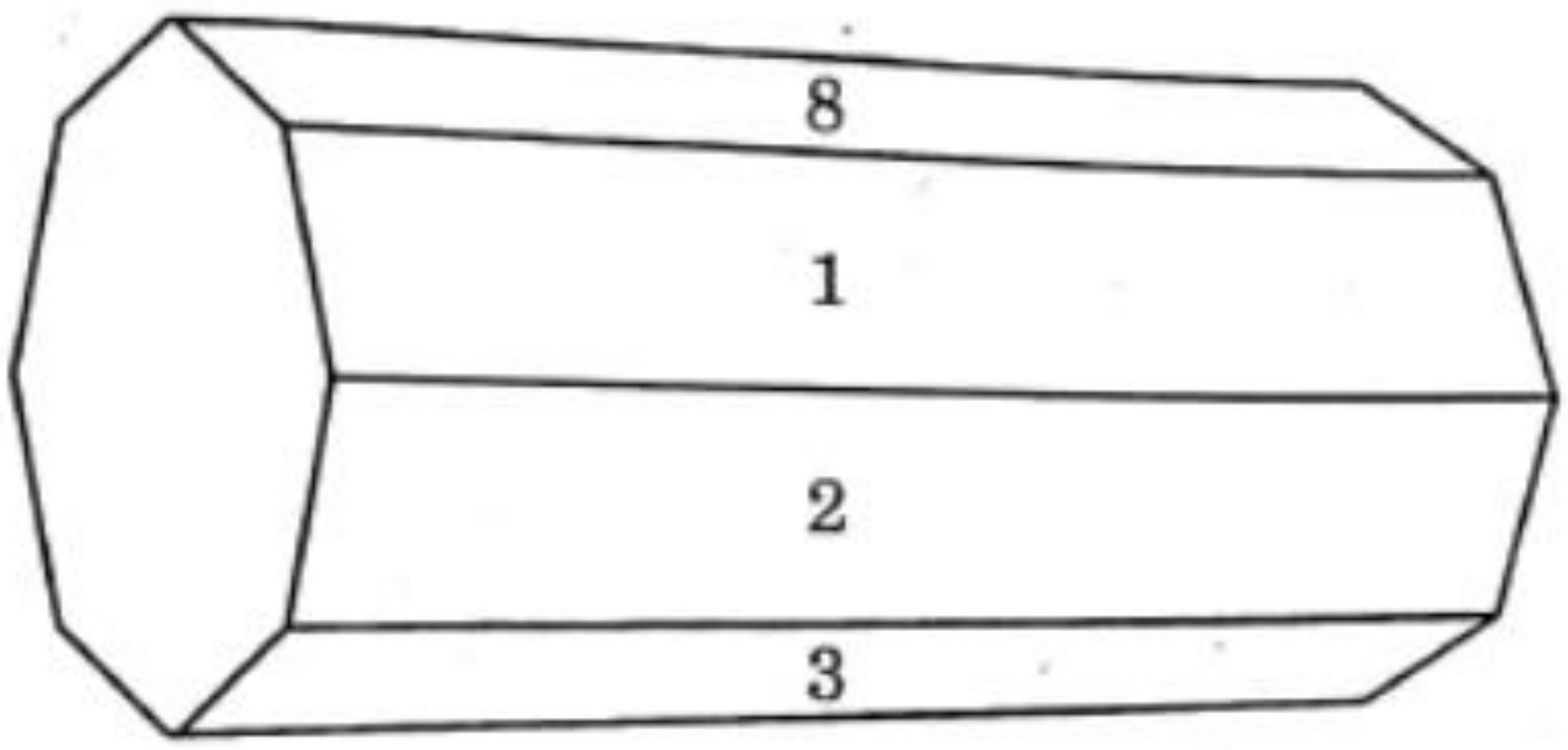
$\Rightarrow x^2 > 10^3$

$\Rightarrow$ for $x > 10\sqrt{10}$ f is ↑
for $0 < x < 10\sqrt{10}$ f is ↓



Pt of minima

37. An octagonal prism is a three-dimensional polyhedron bounded by two octagonal bases and eight rectangular side faces. It has 24 edges and 16 vertices.



The prism is rolled along the rectangular faces and number on the bottom face (touching the ground) is noted. Let X denote the number obtained on the bottom face and the following table give the probability distribution of X.

X :	1	2	3	4	5	6	7	8
P(X) :	p	2p	2p	p	2p	p^2	$2p^2$	$7p^2 + p$

Based on the above information, answer the following questions :

(i) Find the value of p. $p = \frac{1}{10}$ 1

(ii) Find $P(X > 6)$. $P(X>6) = 0.19$ 1

(iii) (a) Find $P(X = 3m)$, where m is a natural number. $= 0.21$ 2

OR

(iii) (b) Find the mean E(X). $= 4.06$ 2

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37 (i) $\Sigma P(x_i) = 1$

$\Rightarrow \quad p + 2p + 2p + p + 2p + p^2 + 2p^2 + 7p^2 + p = 1$

$\Rightarrow \quad 10p^2 + 9p - 1 = 0$

$10p^2 + 10p - p - 1 = 0$

$10p(p+1) - (p+1) = 0$

$p = -1$ or $p = \frac{1}{10}$ Ans.

(ii) $P(X > 6)$

$= P(X=7) + P(X=8)$

$= 2p^2 + 7p^2 + p$

$= 9p^2 + p$

$= \frac{9}{100} + \frac{1}{10}$

$= .09 + 0.1$

$P(X>6) = \boxed{0.19}$ Ans.

(iii) a $P(X = 3m) = P(X=3) + P(X=6)$

$= 2p + p^2$

$= 2 \times \frac{1}{10} + \frac{1}{100}$

$= 0.2 + 0.01 = 0.21$ Ans.

OR

(iii) b Mean $E(X) = \Sigma x_i P(x_i)$

$= p + 2(2p) + 3(2p) + 4p + 10p + 6p^2 + 14p^2 + 56p^2 + 8p$

$= 33p + 76p^2$

$= \frac{33}{10} + \frac{76}{100}$

$= 3.3 + .76 = 4.06$ Ans.

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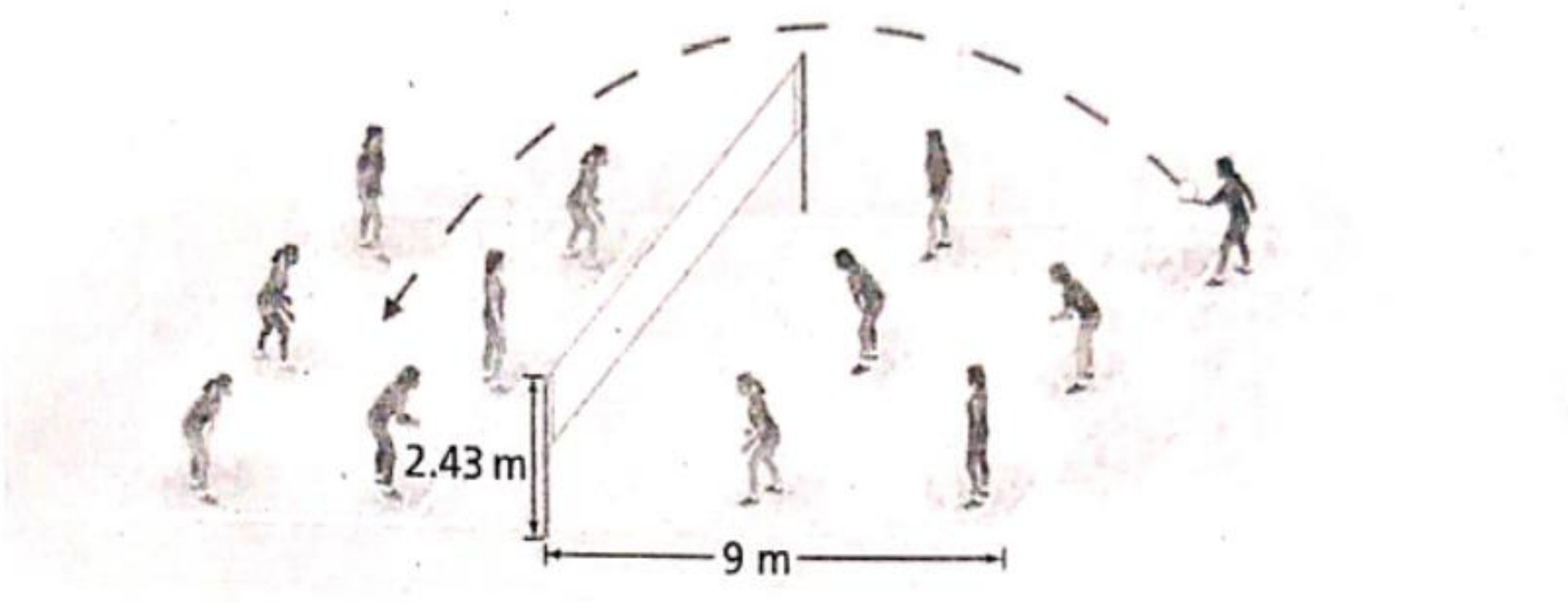
38. A volleyball player serves the ball which takes a parabolic path given by the equation $h(t) = -\frac{7}{2}t^2 + \frac{13}{2}t + 1$, where h(t) is the height of ball at any time t (in seconds), $(t \geq 0)$.



Based on the above information, answer the following questions :

(i) Is h(t) a continuous function ? Justify. Yes, it is poly. function **2**

(ii) Find the time at which the height of the ball is maximum. **2**

~~$t = \frac{13}{14}$ sec~~ $t = \frac{13}{14}$ sec

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38 (i) Yes, it is polynomial function.

(ii) Time when height is maximum

$$h = -\frac{7}{2}t^2 + \frac{13}{2}t + 1$$

$$\frac{dh}{dt} = -7t + \frac{13}{2}, \quad \frac{d^2h}{dt^2} = -7 \quad (\text{maxima})$$

~~for increase~~

$\cancel{d^2h}$

$$\frac{dh}{dt} = 0 \Rightarrow t = \frac{13}{14} \text{ sec} \quad (\text{pt of maxima})$$

So $\boxed{\text{time} = \frac{13}{14} \text{ sec} \quad \text{Ans.}}$