Kota, Rajasthan

CORE ACADEMY

Enhancing ability to learn & Express

A Premier Institute for

Since 2012

JEE | NEET | 8th – 12th | Commerce | CA - Foundation | Humanities

Announces

USEFUL EVENTS

[For 12th Appearing / Passed Students]

MARCH - APRIL 2024

05 TUES

TEST SERIES – NEET 2024

8 Part syllabus, 15 full syllabus test papers with discussion.
From 5th March to 25th April 2024, Any 4 tests are Free.

22 FRIDAY

CRASH COURSE – NEET / CUET

NEET: Assured prep for 550+ marks, week wise schedule
CUET: For Domain Physics, Chemistry, Biology

28 THURS

FREE*

CUET 2024 GUIDANCE SEMINAR

Any doubt regarding CUET 2024? Get knowledge about CUET 2024 in all domains of science, commerce, Arts.

APRIL 03 WED

CRASH COURSE – CUET 2024

For Domains – Accounts, Economics, Business Studies, Political Science, Legal Studies, History, Geography.

[Scan to join or to get a call]

For more details, Call or Visit

Core Academy, 1st & 2nd Floor above Dr. Rakshit Dental Clinic, Mission Hospital Road, Bilaspur

9630588100, 9893783828 | coreacademybsp.in | Core Academy

12th CBSE MATHS SET – 1 CODE 65/3/1

1. If $A = [a_{ij}]$ is an identity matrix, then which of the following is true ?

D

(A) $a_{ij} = \begin{cases} 0, & \text{if } i = j \\ 1, & \text{if } i \neq j \end{cases}$ (B) $a_{ij} = 1, \forall\ i, j$

(C) $a_{ij} = 0, \forall\ i, j$ (D) $a_{ij} = \begin{cases} 0, & \text{if } i \neq j \\ 1, & \text{if } i = j \end{cases}$

2. Let R_+ denote the set of all non-negative real numbers. Then the function $f : R_+ \to R_+$ defined as $f(x) = x^2 + 1$ is :

A

(A) one-one but not onto (B) onto but not one-one

(C) both one-one and onto (D) neither one-one nor onto

3. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be a square matrix such that adj $A = A$. Then, $(a + b + c + d)$ is equal to :

A

(A) 2a (B) 2b

(C) 2c (D) 0

4. A function $f(x) = |1 - x + |x||$ is :

D

(A) discontinuous at $x = 1$ only (B) discontinuous at $x = 0$ only

(C) discontinuous at $x = 0, 1$ (D) continuous everywhere

5. If the sides of a square are decreasing at the rate of 1·5 cm/s, the rate of decrease of its perimeter is :

B

(A) 1·5 cm/s (B) 6 cm/s

(C) 3 cm/s (D) 2·25 cm/s

12th CBSE MATHS SET – 1 CODE 65/3/1

6. $\int_{-a}^{a} f(x)\,dx = 0$, if :

(A) $f(-x) = f(x)$ (B) $f(-x) = -f(x)$

B (C) $f(a-x) = f(x)$ (D) $f(a-x) = -f(x)$

7. $x \log x \frac{dy}{dx} + y = 2 \log x$ is an example of a :

(A) variable separable differential equation.

(B) homogeneous differential equation.

(C) first order linear differential equation.

C (D) differential equation whose degree is not defined.

8. If $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + \hat{j} - \hat{k}$, then $\vec{a}$ and $\vec{b}$ are :

(A) collinear vectors which are not parallel

(B) parallel vectors

(C) perpendicular vectors

C (D) unit vectors

9. If α, β and γ are the angles which a line makes with positive directions of x, y and z axes respectively, then which of the following is ***not*** true ?

(A) $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

(B) $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

(C) $\cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$

D (D) $\cos\alpha + \cos\beta + \cos\gamma = 1$

12th CBSE MATHS SET – 1 CODE 65/3/1

10. The restrictions imposed on decision variables involved in an objective function of a linear programming problem are called :

(A) feasible solutions (B) constraints

B (C) optimal solutions (D) infeasible solutions

11. Let E and F be two events such that $P(E) = 0{\cdot}1$, $P(F) = 0{\cdot}3$, $P(E \cup F) = 0{\cdot}4$, then $P(F \mid E)$ is :

D (A) 0·6 (B) 0·4 (C) 0·5 (D) 0

12. If A and B are two skew symmetric matrices, then (AB + BA) is :

(A) a skew symmetric matrix (B) a symmetric matrix

B (C) a null matrix (D) an identity matrix

13. If $\begin{vmatrix} 1 & 3 & 1 \\ k & 0 & 1 \\ 0 & 0 & 1 \end{vmatrix} = \pm 6$, then the value of k is :

C (A) 2 (B) – 2 (C) ± 2 (D) ∓ 2

14. The derivative of 2^x w.r.t. 3^x is :

(A) $\left(\frac{3}{2}\right)^x \frac{\log 2}{\log 3}$ (B) $\left(\frac{2}{3}\right)^x \frac{\log 3}{\log 2}$

C (C) $\left(\frac{2}{3}\right)^x \frac{\log 2}{\log 3}$ (D) $\left(\frac{3}{2}\right)^x \frac{\log 3}{\log 2}$

15. If $|\vec{a}| = 2$ and $-3 \le k \le 2$, then $|k\vec{a}| \in$:

(A) [– 6, 4] (B) [0, 4]

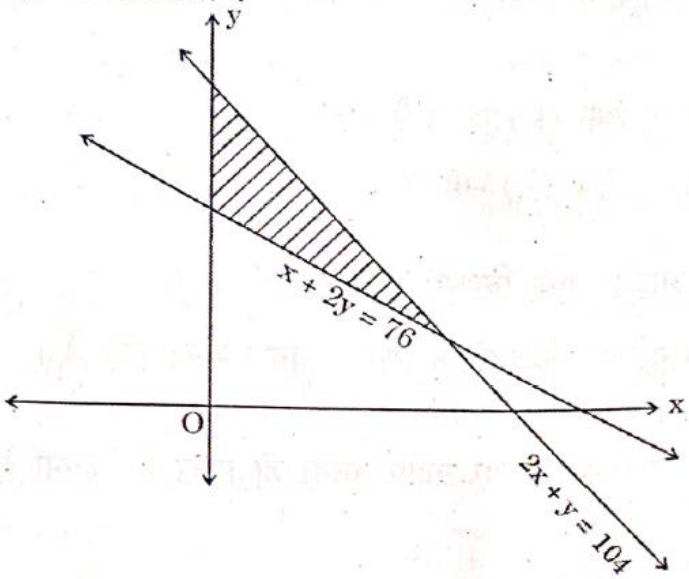
C (C) [4, 6] (D) [0, 6]

12th CBSE MATHS SET – 1 CODE 65/3/1

16. If a line makes an angle of $\frac{\pi}{4}$ with the positive directions of both x-axis and z-axis, then the angle which it makes with the positive direction of y-axis is :

C (A) 0 (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) π

17. Of the following, which group of constraints represents the feasible region given below ?



C (A) $x + 2y \leq 76, 2x + y \geq 104, x, y \geq 0$
(B) $x + 2y \leq 76, 2x + y \leq 104, x, y \geq 0$
(C) $x + 2y \geq 76, 2x + y \leq 104, x, y \geq 0$
(D) $x + 2y \geq 76, 2x + y \geq 104, x, y \geq 0$

12th CBSE MATHS SET – 1 CODE 65/3/1

18. If $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$, then A^{-1} is :

(A) $\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$

(B) $30\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$

A

(C) $\frac{1}{30}\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

(D) $\frac{1}{30}\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$

*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.*

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. *Assertion (A) :* Every scalar matrix is a diagonal matrix.

C *Reason (R) :* In a diagonal matrix, all the diagonal elements are 0.

20. *Assertion (A) :* Projection of $\vec{a}$ on $\vec{b}$ is same as projection of $\vec{b}$ on $\vec{a}$.

D *Reason (R) :* Angle between $\vec{a}$ and $\vec{b}$ is same as angle between $\vec{b}$ and $\vec{a}$ numerically.

12th CBSE MATHS SET – 1 CODE 65/3/1

21. Evaluate :

$$\sec^2\left(\tan^{-1}\frac{1}{2}\right) + \operatorname{cosec}^2\left(\cot^{-1}\frac{1}{3}\right)$$

Q1

$\sec^2\left(\sec^{-1}\frac{\sqrt{5}}{2}\right) + \operatorname{cosec}^2\left(\operatorname{cosec}^{-1}\frac{\sqrt{10}}{3}\right)$

$= \left(\frac{\sqrt{5}}{2}\right)^2 + \left(\frac{\sqrt{10}}{3}\right)^2 = \frac{5}{4} + \frac{10}{9} = \frac{45+40}{36} = \frac{85}{36}$ Ans.

22. (a) If $x = e^{x/y}$, prove that $\frac{dy}{dx} = \frac{\log x - 1}{(\log x)^2}$

OR

(b) Check the differentiability of $f(x) = \begin{cases} x^2+1, & 0 \le x < 1 \\ 3-x, & 1 \le x \le 2 \end{cases}$ at $x = 1$.

$x = e^{x/y}$ PT $\frac{dy}{dx} = \frac{\ln x - 1}{(\ln x)^2}$

$\Rightarrow \frac{x}{y} = \ln x$

$\Rightarrow y = \frac{x}{\ln x}$

$\Rightarrow \frac{dy}{dx} = \frac{\ln x - x\cdot\frac{1}{x}}{(\ln x)^2} = \frac{\ln x - 1}{(\ln x)^2}$ Hence Proved

12th CBSE MATHS SET – 1 CODE 65/3/1

OR

$$f(x) = \begin{cases} x^2+1 & 0 \le x < 1 \\ 3-x & 1 < x \le 2 \end{cases}$$

Check diff at $x = 1$

$$f'(1^-) = \lim_{h \to 0} \frac{f(1-h) - f(1)}{-h}$$

$$= \lim_{h \to 0} \frac{(1-h)^2 + 1 - 2}{-h} = \lim_{h \to 0} \frac{(-2h + h^2)}{-h} = +2$$

$$f'(1^+) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h \to 0} \frac{3-(1+h) - 2}{h} = \lim_{h \to 0} \frac{-h}{h} = -1$$

Since $f'(1^-) \neq f'(1^+) \Rightarrow f$ is not diff at $x = 1$

23. (a) Evaluate :

$$\int_0^{\pi/2} \sin 2x \cos 3x \, dx$$

OR

(b) Given $\frac{d}{dx} F(x) = \frac{1}{\sqrt{2x - x^2}}$ and $F(1) = 0$, find $F(x)$.

12th CBSE MATHS SET – 1 CODE 65/3/1

(a) $\int_0^{\pi/2} \sin 2x \cdot \cos 3x \, dx$

$= \frac{1}{2} \int_0^{\pi/2} (\sin 5x - \sin x) \, dx$

$= \frac{1}{2} \left[\frac{-\cos 5x}{5}\right]_0^{\pi/2} - \frac{1}{2} \left[-\cos x\right]_0^{\pi/2}$

$= \frac{-1}{10} \left(\cos \frac{5\pi}{2} - \cos 0\right) + \frac{1}{2} \left(\cos \frac{\pi}{2} - \cos 0\right)$

$= \frac{1}{10} + \left(\frac{-1}{2}\right) = \frac{1-5}{10} = \frac{-4}{10} = \left(\frac{-2}{5}\right)$ Ans.

OR

(b) $\frac{d}{dx} F(x) = \frac{1}{\sqrt{2x - x^2}}$, $F(1) = 0$, Find $F(x)$

$\Rightarrow F(x) = \int \frac{1}{\sqrt{2x - x^2}} \, dx$

$= \int \frac{dx}{\sqrt{1 - (x-1)^2}}$

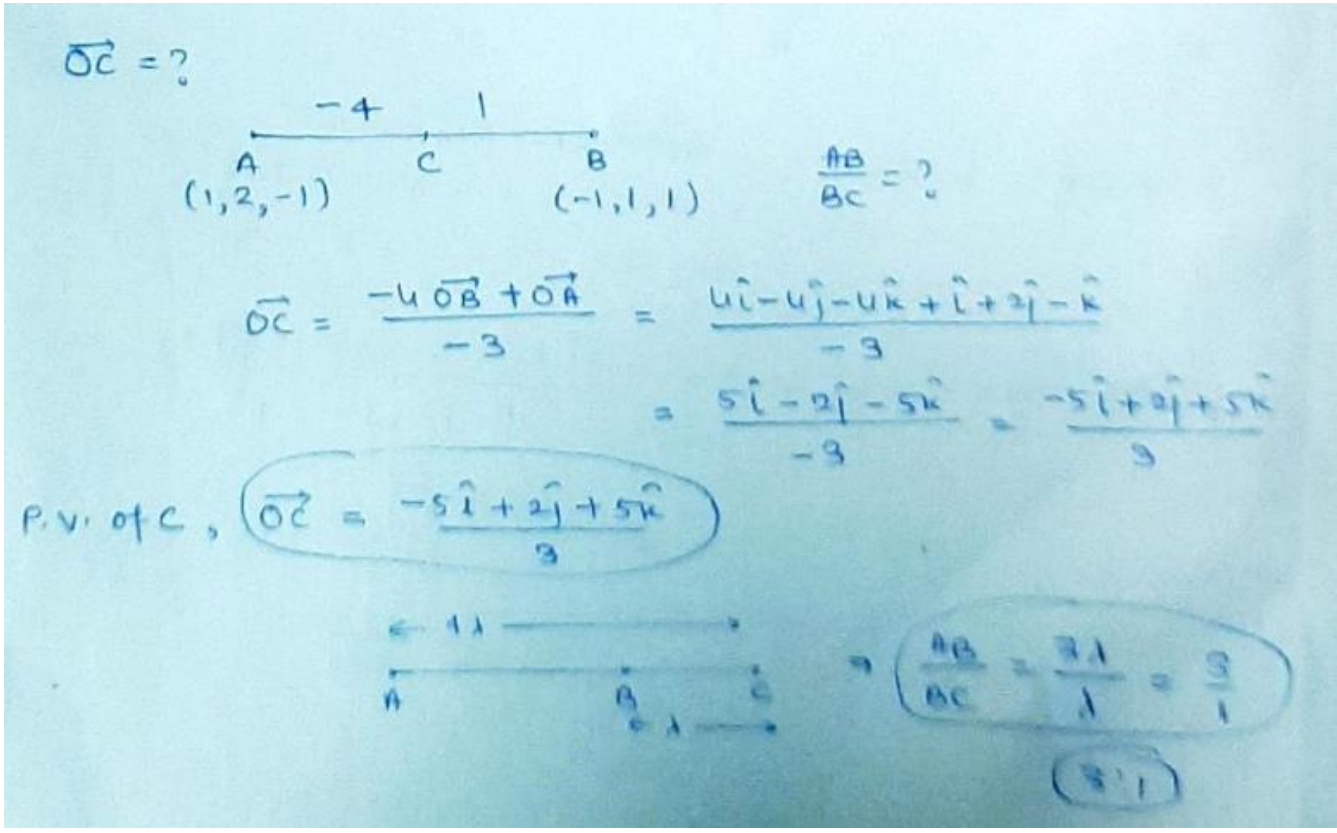
$F(x) = \sin^{-1}(x-1) + C$

$F(1) = 0 = \sin^{-1} 0 + C \Rightarrow (C = 0)$

$\Rightarrow \boxed{F(x) = \sin^{-1}(x-1)}$

12th CBSE MATHS SET – 1 CODE 65/3/1

24. Find the position vector of point C which divides the line segment joining points A and B having position vectors $\hat{i} + 2\hat{j} - \hat{k}$ and $-\hat{i} + \hat{j} + \hat{k}$ respectively in the ratio 4 : 1 externally. Further, find $|\overrightarrow{AB}| : |\overrightarrow{BC}|$.



25. Let $\vec{a}$ and $\vec{b}$ be two non-zero vectors.

Prove that $|\vec{a} \times \vec{b}| \leq |\vec{a}||\vec{b}|$.

State the condition under which equality holds, i.e., $|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}|$.

12th CBSE MATHS SET – 1 CODE 65/3/1

$|\vec{a} \times \vec{b}| = |ab\sin\theta|$

Since $0 \le |\sin\theta| \le 1$

$\Rightarrow \quad 0 \le |ab\sin\theta| \le |ab|$

$\Rightarrow \quad 0 \le |ab\sin\theta| \le |\vec{a}||\vec{b}|$

$\Rightarrow \quad |\vec{a} \times \vec{b}| \le |\vec{a}||\vec{b}|$

when $|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}|$

$|ab\sin\theta| = ab$

$\sin\theta = \pm 1$

$\Rightarrow \theta = \pi/2$ or $-\pi/2$

So $|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}|$ when $\vec{a} \perp \vec{b}$

12th CBSE MATHS SET – 1 CODE 65/3/1

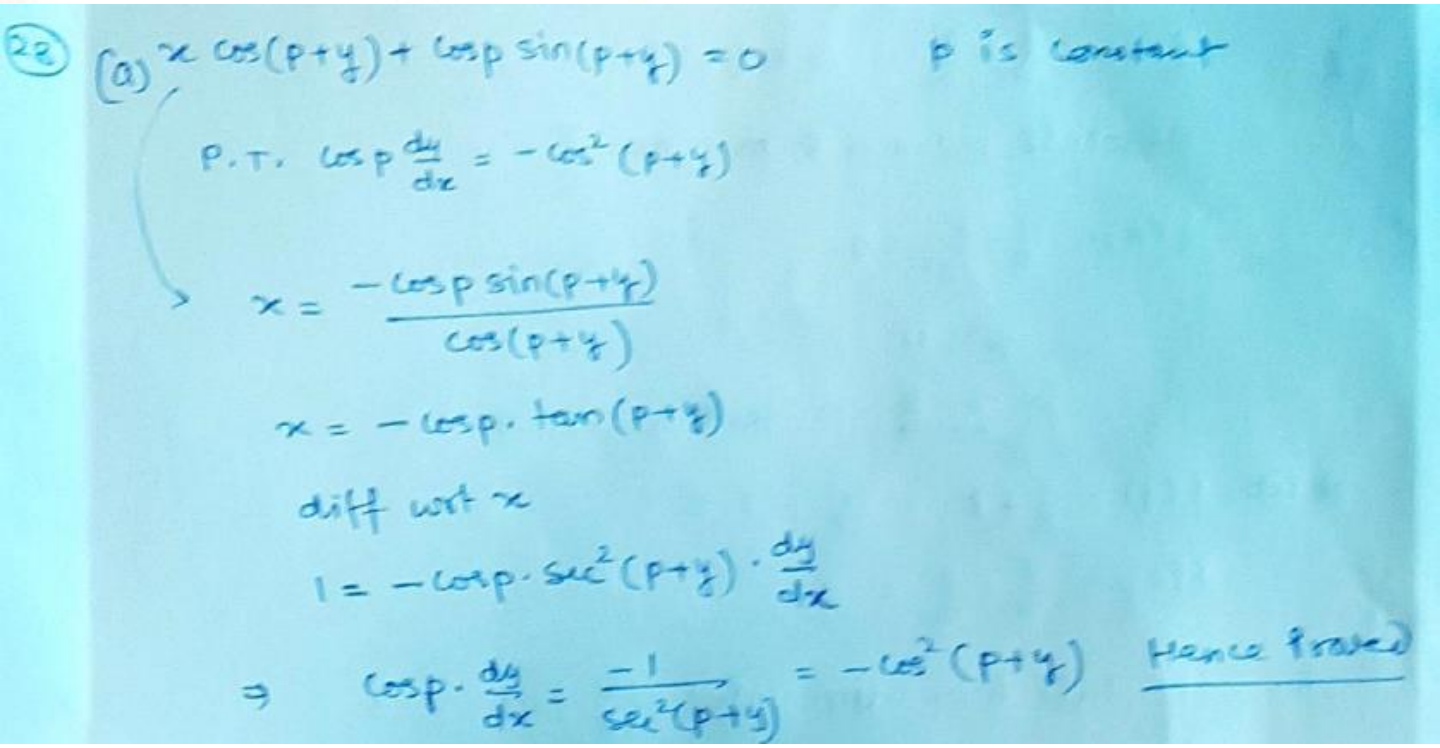
26. (a) If $x \cos(p + y) + \cos p \sin(p + y) = 0$, prove that $\cos p \dfrac{dy}{dx} = -\cos^2(p + y)$, where p is a constant.

OR

(b) Find the value of a and b so that function f defined as :

$$f(x) = \begin{cases} \dfrac{x-2}{|x-2|} + a, & \text{if } x < 2 \\ a + b, & \text{if } x = 2 \\ \dfrac{x-2}{|x-2|} + b, & \text{if } x > 2 \end{cases}$$

is a continuous function.



12th CBSE MATHS SET – 1 CODE 65/3/1

OR

(b) $$f(x) = \begin{cases} \frac{x-2}{|x-2|} + a & x < 2 \\ a+b & x = 2 \\ \frac{x-2}{|x-2|} + b & x > 2 \end{cases}$$

$a = ?$, $b = ?$

f is Continuous

$$\Rightarrow f(x) = \begin{cases} -1+a & x < 2 \\ a+b & x = 2 \\ 1+b & x > 2 \end{cases}$$

$|x-2| = -(x-2)$ if $x < 2$

$f(2^-) = f(2) = f(2^+)$ f is Continuous at $x = 2$

$\Rightarrow a - 1 = a + b = 1 + b$

$\Rightarrow$ $b = -1$, $a = 1$ Ans.

12th CBSE MATHS SET – 1 CODE 65/3/1

27. (a) Find the intervals in which the function $f(x) = \frac{\log x}{x}$ is strictly increasing or strictly decreasing.

OR

(b) Find the absolute maximum and absolute minimum values of the function f given by $f(x) = \frac{x}{2} + \frac{2}{x}$, on the interval $[1, 2]$.

(a) $f(x) = \frac{\ln x}{x}$

$f'(x) = \frac{x \cdot \frac{1}{x} - \ln x}{x^2} = \frac{1 - \ln x}{x^2}$

If f is ↑ then

$f'(x) > 0$

$\frac{1 - \ln x}{x^2} > 0 \Rightarrow 1 - \ln x > 0$

$\ln x - 1 < 0$

$\ln x < 1$

$0 < x < e$

So f is increasing in $(0, e)$ and decreasing in (e, ∞)

Ans

OR

$f(x) = \frac{x}{2} + \frac{2}{x} \quad x \in [1, 2]$

absolute maxima & minima = ?

$f'(x) = \frac{1}{2} - \frac{2}{x^2} = 0$

$\Rightarrow x^2 = 4$

$\Rightarrow x = 2 \Rightarrow$ testing point 1, 2

12th CBSE MATHS SET – 1 CODE 65/3/1

Now $f(1) = \frac{1}{2} + 2 = 5/2 = 2.5$

$f(2) = \frac{2}{2} + \frac{2}{2} = 2$

So absolute maximum value $= \left(\frac{5}{2}\right)$ at $x = 1$

absolute minimum value $= (2)$ at $x = 2$

28. Find :

$$\int \frac{x^2+1}{(x^2+2)(x^2+4)}\,dx$$

$$\int \frac{x^2+1}{(x^2+2)(x^2+4)}\,dx$$

$$= \int \left(\frac{-1/2}{x^2+2} + \frac{3/2}{x^2+4}\right) dx$$

$$= \frac{3}{2}\cdot\frac{1}{2}\tan^{-1}\frac{x}{2} - \frac{1}{2}\cdot\frac{1}{\sqrt{2}}\tan^{-1}\frac{x}{\sqrt{2}} + C$$

$$= \frac{3}{4}\tan^{-1}\frac{x}{2} - \frac{1}{2\sqrt{2}}\tan^{-1}\frac{x}{\sqrt{2}} + C$$

12th CBSE MATHS SET – 1 CODE 65/3/1

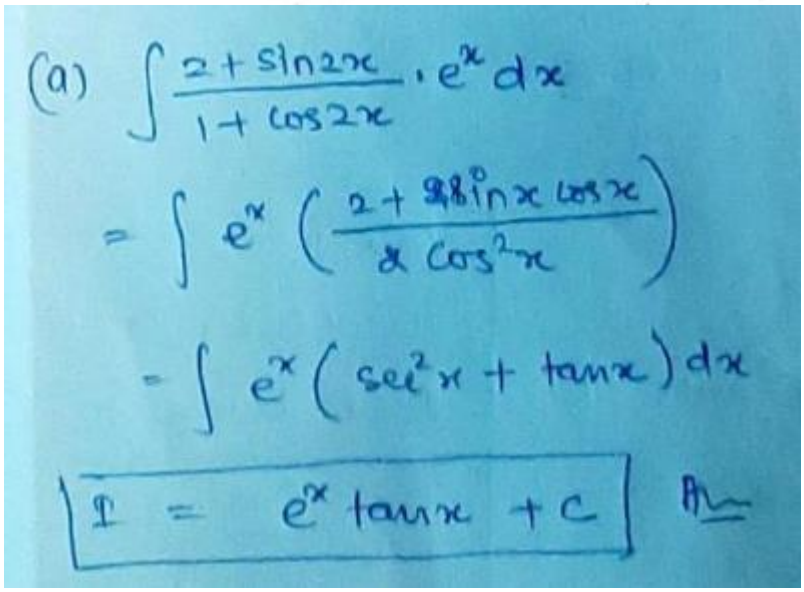
29. (a) Find :

$$\int \frac{2+\sin 2x}{1+\cos 2x} e^x \, dx$$

OR

(b) Evaluate :

$$\int_0^{\pi/4} \frac{1}{\sin x + \cos x} \, dx$$



12th CBSE MATHS SET – 1 CODE 65/3/1

(b) OR

$$I = \int_0^{\pi/4} \frac{1}{\sin x + \cos x}\, dx$$

$$I = \int_0^{\pi/4} \frac{dx}{\sqrt{2}\sin\left(x + \pi/4\right)}$$

$\int \operatorname{cosec} x\, dx = \ln(\operatorname{cosec} x - \cot x)$

$$= \frac{1}{\sqrt{2}} \int_0^{\pi/4} \operatorname{cosec}\left(x + \pi/4\right) dx$$

$$= \frac{1}{\sqrt{2}} \left[\log\left(\operatorname{cosec}\left(x + \pi/4\right) - \cot\left(x + \pi/4\right)\right)\right]_0^{\pi/4}$$

$$= \frac{1}{\sqrt{2}} \left[\ln\left(\operatorname{cosec}\frac{\pi}{2} - \cot\frac{\pi}{2}\right) - \ln\left(\operatorname{cosec}\pi/4 - \cot\pi/4\right)\right]$$

$$= \frac{1}{\sqrt{2}} \left[\ln(1 - 0) - \ln\left(\sqrt{2} - 1\right)\right]$$

$$= \frac{1}{\sqrt{2}} \left[\ln \frac{1}{\sqrt{2}-1}\right]$$

$$\boxed{I = \frac{1}{\sqrt{2}} \ln\left(\sqrt{2} + 1\right)}$$ Ans

30. Solve the following linear programming problem graphically :

Maximise $z = 4x + 3y$,

subject to the constraints

$$x + y \leq 800$$

$$2x + y \leq 1000$$

$$x \leq 400$$

$$x, y \geq 0.$$

12th CBSE MATHS SET – 1 CODE 65/3/1

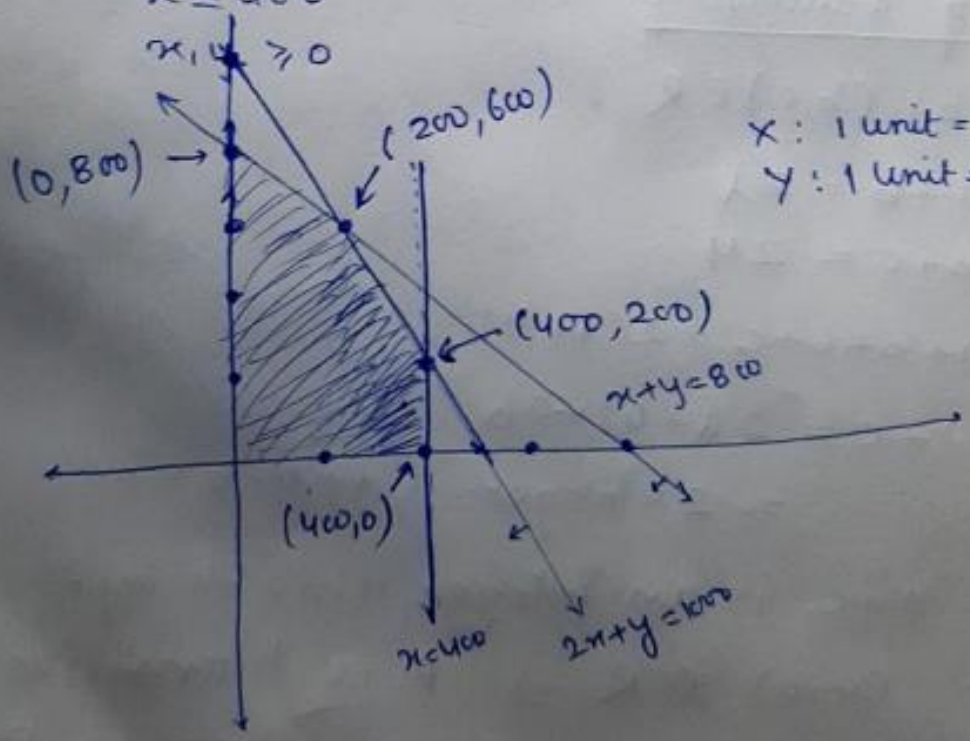
Q. 30 Maximize $Z = 4x + 3y$

$x + y \leq 800$

$2x + y \leq 1000$

$x \leq 400$

$x, y \geq 0$



Corner Point	$Z = 4x + 3y$
(400, 0)	1600
(400, 200)	2200
(200, 600)	2600 ← Maximum
(0, 800)	2400

$Z_{max} = 2600$ Ans

12th CBSE MATHS SET – 1 CODE 65/3/1

31. The chances of P, Q and R getting selected as CEO of a company are in the ratio 4 : 1 : 2 respectively. The probabilities for the company to increase its profits from the previous year under the new CEO, P, Q or R are 0·3, 0·8 and 0·5 respectively. If the company increased the profits from the previous year, find the probability that it is due to the appointment of R as CEO.

$P(P \text{ is selected}) = \frac{4K}{7}$

$P(Q \text{ ——— }) = \frac{K}{7}$

$P(R \text{ ——— }) = \frac{2K}{7}$

$P(\text{profit increased}/P \text{ is selected}) = 0.3$

$P(\text{ ——— }/Q \text{ ——— }) = 0.8$

$P(\text{ ——— }/R \text{ ——— }) = 0.5$

$P(\text{Profit is increased}) = $

$= 4K \times 0.3 + K \times 0.8 + 2K \times 0.5$

$= 1.2K + 0.8K + K = 3K$

$P(R \text{ is selected}/\text{Profit is } \uparrow) = \frac{P(R \text{ is selected \& profit } \uparrow)}{P(\text{Profit is } \uparrow)}$

$= \frac{2K(0.5)}{3K}$

$= \frac{1}{3}$ Ans.

12th CBSE MATHS SET – 1 CODE 65/3/1

32. A relation R on set A = {– 4, – 3, – 2, – 1, 0, 1, 2, 3, 4} be defined as R = {(x, y) : x + y is an integer divisible by 2}. Show that R is an equivalence relation. Also, write the equivalence class [2].

$A = \{-4,-3,-2,-1,0,1,2,3,4\}$

$R = \{(x,y) : x+y \text{ divisible by } 2\}$

Checking reflexive

$\forall\ a \in A$

$a+a = 2a \Rightarrow 2a$ is divisible by 2

$\Rightarrow (a,a) \in R \Rightarrow$ R is reflexive

Checking Symmetry

$\forall\ (a,b) \in R$

$a+b = 2k$

$\Rightarrow b+a = 2k$

$\Rightarrow (b,a) \in R \Rightarrow$ R is symmetric

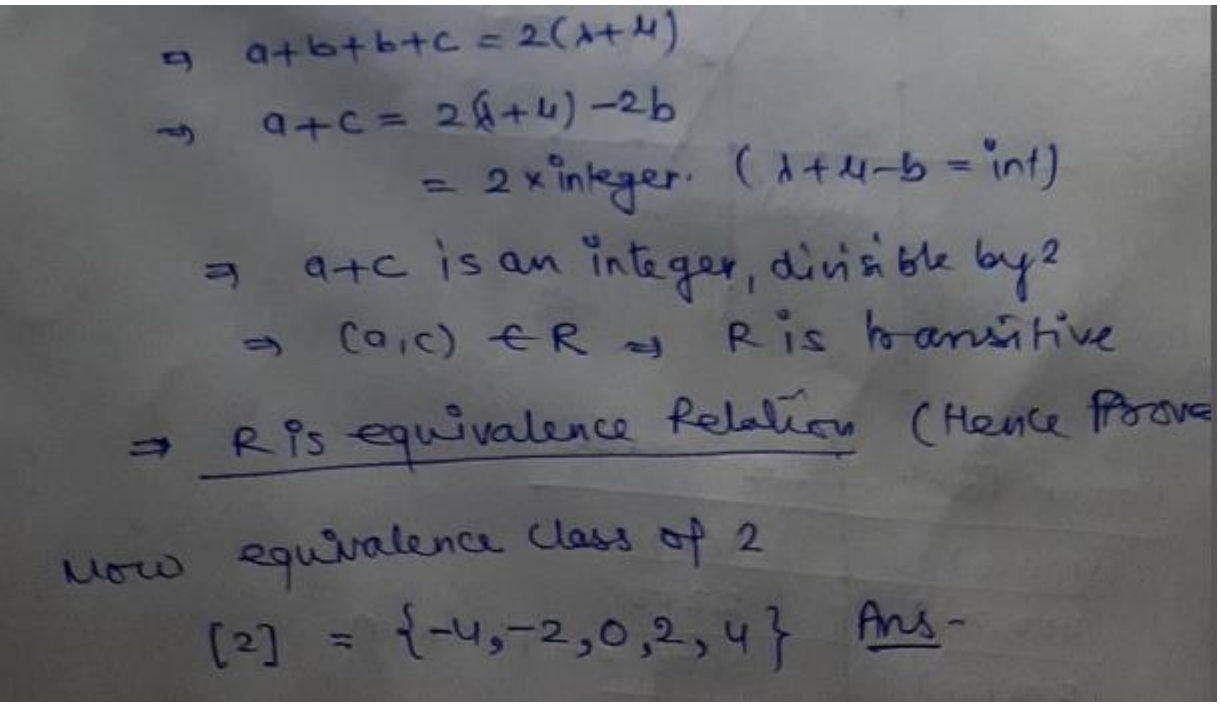
Checking transitivity

$\forall\ (a,b), (b,c) \in R$

$a+b = 2\lambda$

$b+c = 2\mu$

12th CBSE MATHS SET – 1 CODE 65/3/1



33. (a) It is given that function $f(x) = x^4 - 62x^2 + ax + 9$ attains local maximum value at $x = 1$. Find the value of 'a', hence obtain all other points where the given function $f(x)$ attains local maximum or local minimum values.

OR

(b) The perimeter of a rectangular metallic sheet is 300 cm. It is rolled along one of its sides to form a cylinder. Find the dimensions of the rectangular sheet so that volume of cylinder so formed is maximum.

12th CBSE MATHS SET – 1 CODE 65/3/1

(a) $f(x) = x^4 - 62x^2 + ax + 9$

local max at $x = 1$, $a = ?$

Points of Local Maximum, Local min.

$f'(1) = 0$ (f has local max. at $x = 1$)

$4x^3 - 124x + a = 0$

$4 - 124 + a = 0$

$a = 120$ Ans

So $f(x) = x^4 - 62x^2 + 120x + 9$

$f'(x) = 4x^3 - 124x + 120 = 0$

$\Rightarrow x^3 - 31x + 30 = 0$

$\Rightarrow (x-1)(x^2 + x - 30) = 0$

$\Rightarrow (x-1)(x+6)(x-5) = 0$

$x = 1$ Point of local maximum (given)

at $x = 5$

12th CBSE MATHS SET – 1 CODE 65/3/1

$f''(x) = 4(3x^2 - 31)$

$f''(5) = 4(75 - 31) = +$ $\Rightarrow$ $x = 5$ is point of local minima

at $x = -6$

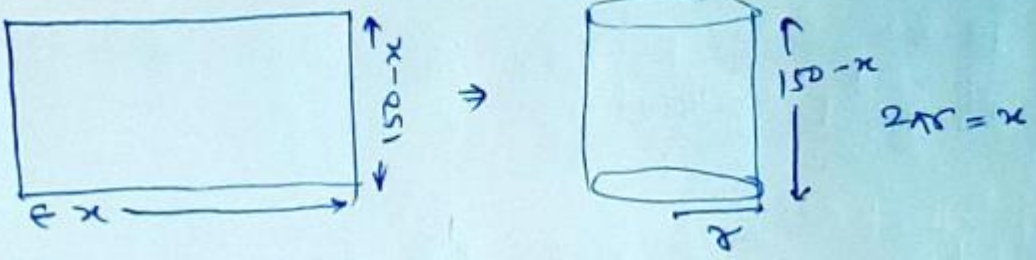
$f''(-6) = 4(3 \cdot 36 - 31) = +$

$\Rightarrow$ $x = -6$ is point of local minimum

So Points of local maximum : $x = 1$

Points of local minimum : $x = 5, \; x = -6$

OR



Perimeter = 300 cm

Let length = x, breadth = $150 - x$

Volume of cyl = $V = \pi r^2 h$

12th CBSE MATHS SET – 1 CODE 65/3/1

$= \pi\left(\frac{x}{2\pi}\right)^2 \cdot (150 - x)$

$= \frac{1}{4\pi} x^2 (150 - x) = \frac{1}{4\pi}(150x^2 - x^3)$

for V is maximum $\frac{dV}{dx} = 0$

$\Rightarrow 300x - 3x^2 = 0 \Rightarrow x^2 = 100$

$x = 100$ or ~~0~~

~~See~~ $\frac{d^2V}{dx^2} = \frac{1}{4\pi}(300 - 6x)\Big)_{x=100} = -ve$

$\Rightarrow$ Vol. is max. at $x = 100$

So length = 100 cm, breadth = 50 cm. Ans.

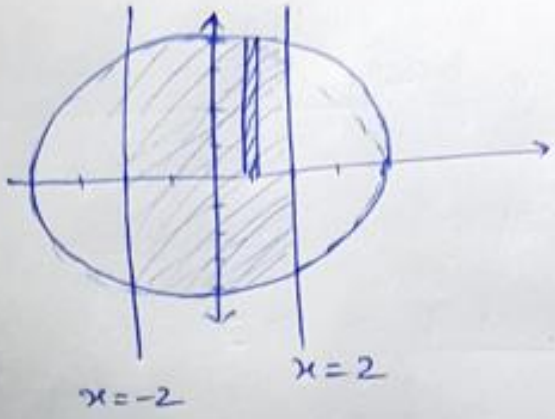
dimensions

12th CBSE MATHS SET – 1 CODE 65/3/1

34. Using integration, find the area of the region enclosed between the circle $x^2 + y^2 = 16$ and the lines $x = -2$ and $x = 2$.

$x^2+y^2=16$

$x=-2, \quad x=2.$



Area of shaded region $= 2\int_{-2}^{2} \sqrt{16-x^2}\,dx$

$= 4\int_{0}^{2} \sqrt{16-x^2}\,dx$

$= 4\left[\frac{x}{2}\sqrt{16-x^2} + \frac{16}{2}\sin^{-1}\frac{x}{4}\right]_0^2$

$= 4\left[\left(\sqrt{16-4} + 8\sin^{-1}\frac{1}{2}\right) - 0\right]$

$= 4\left[2\sqrt{3} + \frac{4\pi}{3}\right] = 8\sqrt{3} + \frac{16\pi}{3}$ sq. units

12th CBSE MATHS SET – 1 CODE 65/3/1

35. (a) Find the equation of the line passing through the point of intersection of the lines $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ and $\frac{x-1}{0} = \frac{y}{-3} = \frac{z-7}{2}$ and perpendicular to these given lines.

OR

(b) Two vertices of the parallelogram ABCD are given as A(–1, 2, 1) and B(1, –2, 5). If the equation of the line passing through C and D is $\frac{x-4}{1} = \frac{y+7}{-2} = \frac{z-8}{2}$, then find the distance between sides AB

given lines $L_1: \frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3} = \lambda \Rightarrow$ drs = 1, 2, 3

$L_2: \frac{x-1}{0} = \frac{y}{-3} = \frac{z-7}{2} = \mu \Rightarrow$ drs = 0, –3, 2

any Point on $L_1 = (\lambda, 2\lambda+1, 3\lambda+2)$

$L_2 = (1, -3\mu, 2\mu+7)$

Point of intersection $(\lambda, 2\lambda+1, 3\lambda+2) = (1, -3\mu, 2\mu+7)$

$\Rightarrow \lambda = 1$, $\mu = -1$

So point of intersection = (1, 3, 5)

drs of line ⊥ to L_1 & L_2 :

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 0 & -3 & 2 \end{vmatrix}$$

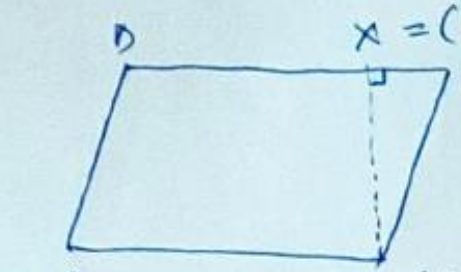
$= \hat{i}(4+9) - \hat{j}(2) + \hat{k}(-3)$

$= 13\hat{i} - 2\hat{j} - 3\hat{k} \equiv$ (13, –2, –3)

So required line $\frac{x-1}{13} = \frac{y-3}{-2} = \frac{z-5}{-3}$ Ans

12th CBSE MATHS SET – 1 CODE 65/3/1

OR
b.



$X = (\lambda+4, -2\lambda-7, 2\lambda+8)$

Eq^n of line CD : $\frac{x-4}{1} = \frac{y+7}{-2} = \frac{z-8}{2} = \lambda$

distance b/w AB & CD = BX

$X = (\lambda+4, -2\lambda-7, 2\lambda+8)$

drs of BX = $\lambda+3, -2\lambda-5, 2\lambda+3$

drs of CD = $1, -2, 2$

$BX \perp CD \Rightarrow (\lambda+3) - 2(-2\lambda-5) + 2(2\lambda+3) = 0$

$\Rightarrow \lambda+3+4\lambda+10+4\lambda+6 = 0$

$\Rightarrow 9\lambda + 19 = 0$

12th CBSE MATHS SET – 1 CODE 65/3/1

$$\lambda = \frac{-19}{9}$$

So $BX = \left(\frac{-19}{9}+4, \frac{38}{9}-7, \frac{-38}{9}+8\right)$

$$= \left(+\frac{17}{9}, -\frac{25}{9}, \frac{34}{9}\right)$$

$$BX = \sqrt{\left(\frac{-19}{9}+3\right)^2 + \left(\frac{38}{9}-5\right)^2 + \left(\frac{-38}{9}+3\right)^2}$$

$$= \sqrt{\frac{64}{81} + \frac{49}{81} + \frac{121}{81}}$$

$$BX = \sqrt{\frac{234}{81}} = \frac{\sqrt{234}}{9}$$ Ans.

Area of $\triangle ABX = \frac{1}{2} \times AB \times BX$

$$= 2 \times \frac{\sqrt{234}}{9} \times \frac{1}{3}$$ Ans.

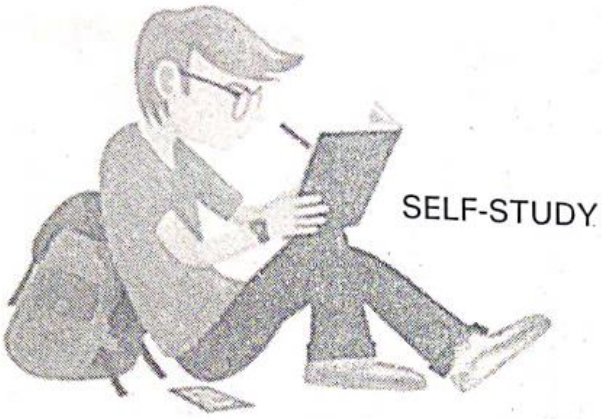
$$= \frac{2}{3}\sqrt{234} \text{ Sq. units}$$

AB =

12th CBSE MATHS SET – 1 CODE 65/3/1

Case Study – 1

36. Self-study helps students to build confidence in learning. It boosts the self-esteem of the learners. Recent surveys suggested that close to 50% learners were self-taught using internet resources and upskilled themselves.



A student may spend 1 hour to 6 hours in a day in upskilling self. The probability distribution of the number of hours spent by a student is given below :

$$P(X = x) = \begin{cases} kx^2, & \text{for } x = 1, 2, 3 \\ 2kx, & \text{for } x = 4, 5, 6 \\ 0, & \text{otherwise} \end{cases}$$

where x denotes the number of hours.

Based on the above information, answer the following questions :

(i) Express the probability distribution given above in the form of a probability distribution table. *1*

(ii) Find the value of k. *1*

(iii) (a) Find the mean number of hours spent by the student. *2*

OR

(iii) (b) Find $P(1 < X < 6)$. *2*

12th CBSE MATHS SET – 1 CODE 65/3/1

$$P(X=x) = \begin{cases} kx^2 & x = 1,2,3 \\ 2kx & x = 4,5,6 \\ 0 & \text{otherwise} \end{cases}$$

x = no. of hrs.

(i) Prob. distribution

x	$P(x)$
1	k
2	$4k$
3	$9k$
4	$8k$
5	$10k$
6	$12k$
Σ =	

X = no of hours.
$P(X)$ = Prob (studying X hours)

(ii) $\Sigma P(x) = 1$

$\Rightarrow 43k = 1 \Rightarrow \boxed{k = \frac{1}{43}}$ Ans

(iii) (a) $E(X) = \Sigma x_i P(x_i)$

$= k + 8k + 27k + 32k + 50k + 72k$

$= 190k$

$\boxed{E(X) = \frac{190}{43} = \text{mean}}$ Ans. $\frac{190}{43}$ hrs.

12th CBSE MATHS SET – 1 CODE 65/3/1

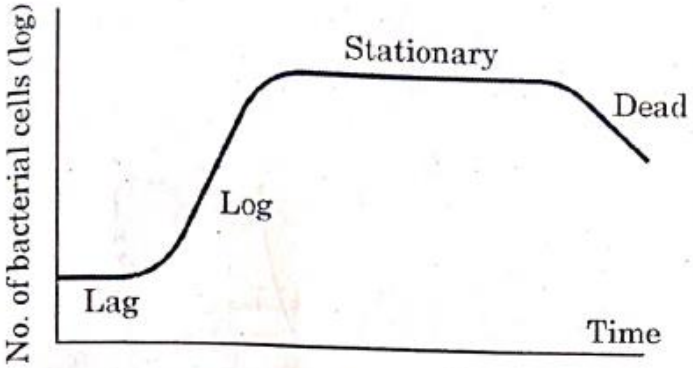
(iii) (b) $P(1<X<6) = P(X=2) + P(X=3) + P(X=4) + P(X=5)$

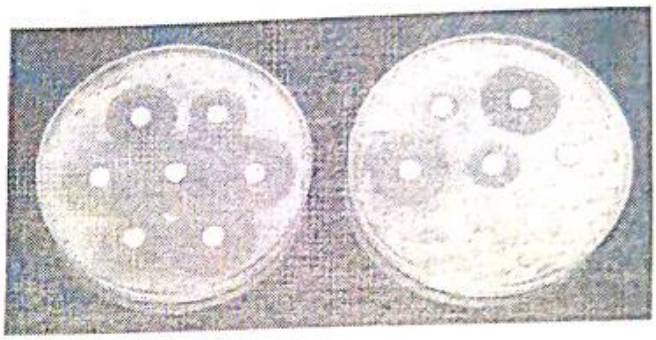
$= 4k + 9k + 8k + 10k$

$= 31k$

$= \frac{31}{43}$ Ans.

37. A bacteria sample of certain number of bacteria is observed to grow exponentially in a given amount of time. Using exponential growth model, the rate of growth of this sample of bacteria is calculated.





The differential equation representing the growth of bacteria is given as :

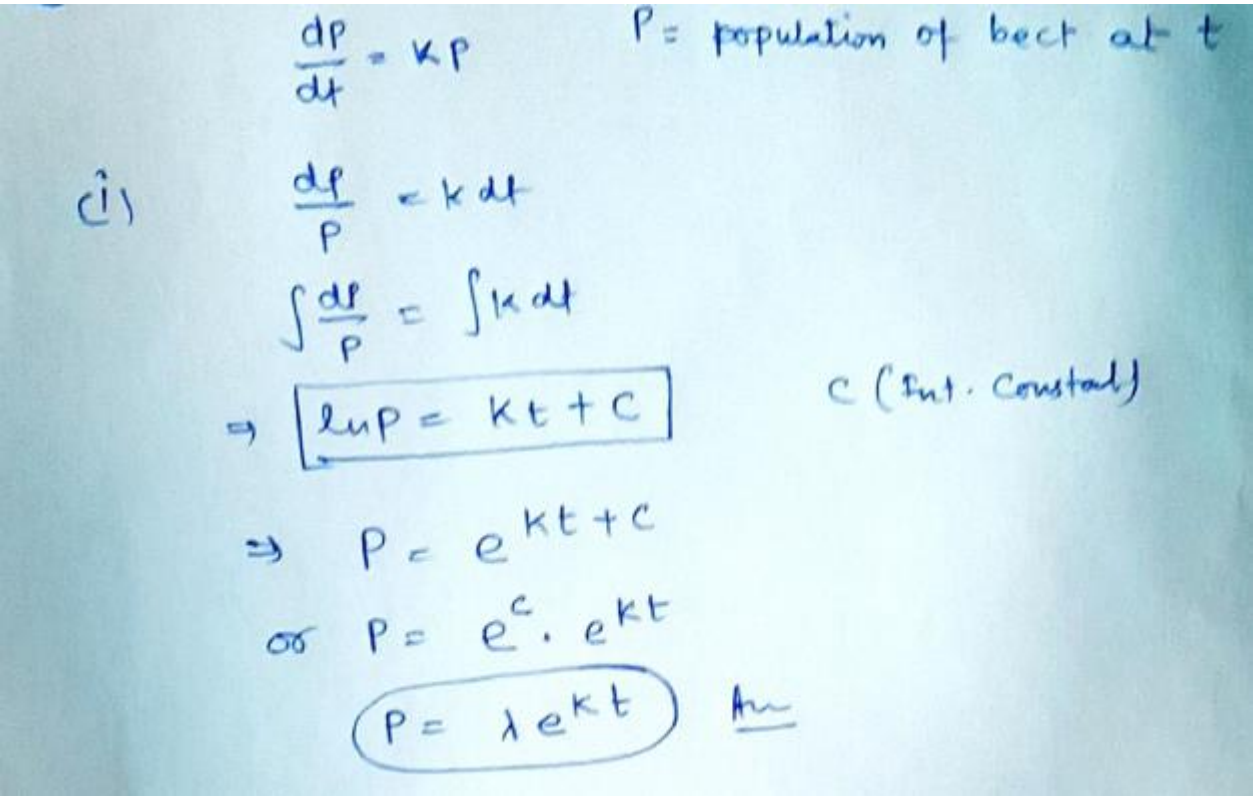
$\frac{dP}{dt} = kP$, where P is the population of bacteria at any time 't'.

Based on the above information, answer the following questions :

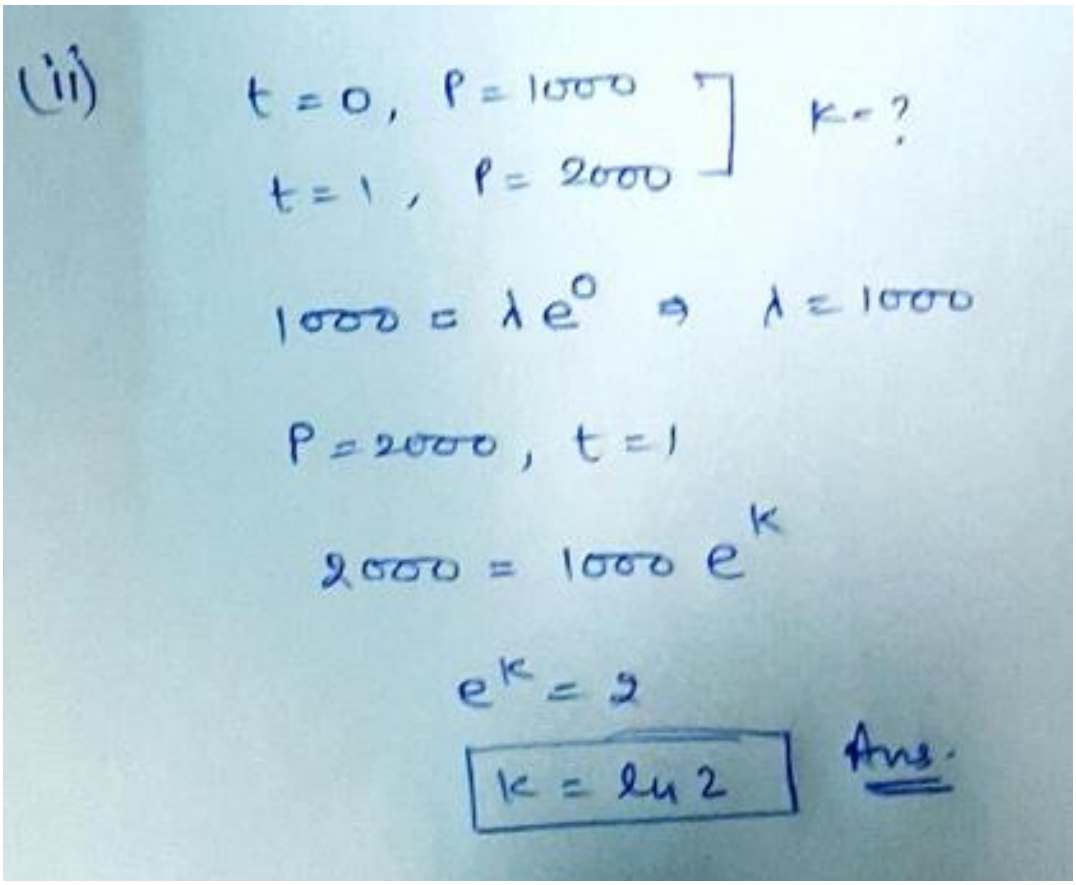
12th CBSE MATHS SET – 1 CODE 65/3/1

(i) Obtain the general solution of the given differential equation and express it as an exponential function of 't'. 2

(ii) If population of bacteria is 1000 at t = 0, and 2000 at t = 1, find the value of k. 2

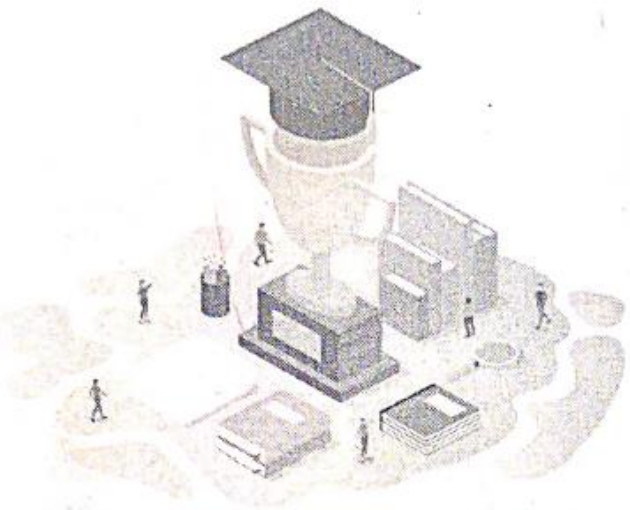


12th CBSE MATHS SET – 1 CODE 65/3/1



12th CBSE MATHS SET – 1 CODE 65/3/1

38. A scholarship is a sum of money provided to a student to help him or her pay for education. Some students are granted scholarships based on their academic achievements, while others are rewarded based on their financial needs.



Every year a school offers scholarships to girl children and meritorious achievers based on certain criteria. In the session 2022 – 23, the school offered monthly scholarship of ₹ 3,000 each to some girl students and ₹ 4,000 each to meritorious achievers in academics as well as sports.

In all, 50 students were given the scholarships and monthly expenditure incurred by the school on scholarships was ₹ 1,80,000.

Based on the above information, answer the following questions :

(i) Express the given information algebraically using matrices. *1*

(ii) Check whether the system of matrix equations so obtained is consistent or not. *1*

(iii) (a) Find the number of scholarships of each kind given by the school, using matrices. *2*

OR

(iii) (b) Had the amount of scholarship given to each girl child and meritorious student been interchanged, what would be the monthly expenditure incurred by the school ? *2*

12th CBSE MATHS SET – 1 CODE 65/3/1

Types of scholarships – Girl Candidate (Rs. 3000)
Meritorious student (Rs. 4000)

Total no of students = 50

Let no. of students (getting Rs. 3000/month) = x
——— (——— 4000/month) = y

ATQ.

(i) $x + y = 50$
$3000x + 4000y = 1,80,000$
$\Rightarrow \begin{pmatrix} 1 & 1 \\ 3000 & 4000 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 50 \\ 1,80,000 \end{pmatrix}$

(ii) in (i) let system is $AX = B$

here $|A| \neq 0 \Rightarrow$ system is consistent

12th CBSE MATHS SET – 1 CODE 65/3/1

(iii)

$$X = A^{-1}B$$

$$A^{-1} = \frac{1}{|A|}\,\text{Adj}(A) = \frac{1}{1000}\begin{pmatrix} 4000 & -1 \\ -3000 & 1 \end{pmatrix}$$

$$X = \frac{1}{1000}\begin{pmatrix} 4000 & -1 \\ -3000 & 1 \end{pmatrix}\begin{pmatrix} 50 \\ 180000 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{1000}\begin{pmatrix} 200000 - 180000 \\ -150000 + 180000 \end{pmatrix} = \begin{pmatrix} 20 \\ 30 \end{pmatrix}$$

$x = 20,\ y = 30$

no of students getting Scho. Rs. 3000/mon = 20
4000/mon = 30 Ans

(iii) OR (b) on interchanging

expenses incurred = $30 \times 3000 + 20 \times 4000$
$= 90000 + 80000$
$= 1,70,000$ Ans

Kota, Rajasthan

CORE ACADEMY

Enhancing ability to learn & Express

A Premier Institute for

Since 2012

JEE | NEET | 8th – 12th | Commerce | CA - Foundation | Humanities

Attention! Class 12th Appearing/Passed Students

Top Leading Institute in **BILASPUR** Zone since 2012 ...

Bilaspur Zone's NO.1 Institute since 2013

Announces....

COURSES FOR YOUR NEED

CRASH COURSE

FOR NEET / CUET 2024

In 42 days, get assured preparation for marks **550+**. We have week wise plan with test series accordingly.

22nd MARCH 2024

- हिन्दी माध्यम के लिए पृथक बैचेज

TEST SERIES JEE / NEET

FOR NEET / JEE 2024

NEET: 25 MOCK TESTS, OFFLINE (English and Hindi Medium)

JEE MAIN: 40 MOCK TESTS, ONLINE (Only English medium)

MARCH, APRIL 2024

- At ₹1500 only, **4 tests are FREE for all**

FREE* FREE* FREE* Useful Tools for You FREE* FREE* FREE*

1. NCERT **line by line questions** (50 – 100 Qs from each chapter) for NEET.
2. Test Series for JEE MAIN / ADV / NEET 2024 With discussion (4 Free mock tests for all).
3. CBSE 12th Board 2024 Q. papers solution on same day (provided on WhatsApp or our you tube channel.

Free*

To get these, scan this code OR visit Academy or call

coreacademybsp.in/enquiry

Our Best Faculty for Your Crash Course

OMESH RENWAL
Director, Exp. 13 Years
B.Tech. - UCE, KOTA, RAJ.

ROHIT SHUKLA
Exp. 8 Years
M. Sc. (Physics), GGU

PRADIP KATHROTIYA
Exp. 7 Years
B. TECH. NIT DELHI

Dr. PRABHAKAR SHRIVAS
Exp. 9 Years
Ph. D. VNIT NAGPUR

DEEPAK KAUSHIK
Exp. 15 Years
B. E. CHEMICAL (GGU)

Dr. SWARNAV BOSE
Exp. 2 Year
B. D. S. Triveni Institute, BSP

MANMOHAN SAHU
Exp. 3 Year
M. Sc., CTET

Be the part of Bilaspur zone's No. 1 Institute since 2013

Enroll Till 19th March 2024 & Get Flat 20.0% off

For more details, Call or Visit

Core Academy, 1st & 2nd Floor above Dr. Rakshit Dental Clinic, Mission Hospital Road, Bilaspur

9630588100, 9893783828 coreacademybsp.in Core Academy

Kota, Rajasthan

CORE ACADEMY

Enhancing ability to learn & Express

A Premier Institute for

Since 2012

JEE | NEET | 8th – 12th | Commerce | CA - Foundation | Humanities

Success Stories : JEE (Last 3 Years)

2024

99.31 (Percentile)

GAURAV BAIRAGI
1.5 Yr. Classroom

98.79 (Percentile)

AMISHA RAO
1 Yr. Classroom

97.67 (Percentile)

NAYAN PAL
2 Yr. Classroom

93.04 (Percentile)

DEEP JYOTISHI
2 Yr. Classroom

92.09 (Percentile)

AYUSH AGRAWAL
3 Yr. Classroom

2023

98.13 (Percentile)

NIT TRICHI*
GAURAV BAIRAGI
1 Yr. Classroom

2169 (AIR)

IIT KHARAGPUR
ADITYA P. SINGH
4 Yr. Classroom

96.12 (Percentile)

NIT RAIPUR
MAHIMA SHUKLA
1 Yr. Classroom

93.51 (Percentile)

NIT RAIPUR
MAYANK K GUPTA
2 Yr. Classroom

4 / 27
SELECTION RATIO
Got NIT or IIT

2022

99.44 (Percentile)

NIT SURATHKAL
SANIDDHYA SINGH
4 Yr. Classroom

99.40 (Percentile)

NIT ROURKELA
SWATI KATAILIHA
2 Yr. Classroom

98.20 (Percentile)

NIT ROURKELA
KUNAL SHARMA
3 Yr. Classroom

96.37 (Percentile)

NIT RAIPUR
PRAKHAR SHRIVASTAVA
3 Yr. Classroom

96.12 (Percentile)

VIT VELLORE
RADHIKA CHOPDE
2 Yr. Classroom

8 / 30
SELECTION RATIO
Got NIT or IIT

Success Stories : NEET (Last 2 Years)

MBBS 1st Attempt 2023

NANDINI PATRA
4 Yr. Classroom
GMC, JAGDALPUR

MBBS 1st Attempt 2023

PALAK KAUSHIK
4 Yr. Classroom
GMC, KANKER

MBBS 1st Attempt 2023

AMIT K JAGAT
Short Term Course
GMC, RAIPUR

MBBS 1st Attempt 2022

ANANYA SHRIVASTAVA
Short Term Course
GMC, KANKER

MBBS 1st Attempt 2022

AYUSHI DUBEY
3 Yr. Classroom
GMC, KORBA

MBBS 1st Attempt 2022

ANVITAA MURTY
4 Yr. Classroom
IMS, Bhubaneswar

MBBS 1st Attempt 2022

SIDDHI AGRAWAL
4 Yr. Classroom
YENEPOYA, MANGLORE

MBBS 1st Attempt 2022

PRIYAL YADAV
4 Yr. Classroom
RIMS, RAIPUR

SELECTION RATIO 2023
03 / 43
Got MBBS

SELECTION RATIO 2022
05 / 24
Got MBBS

For more details, Call or Visit

Core Academy, 1st & 2nd Floor above Dr. Rakshit Dental Clinic, Mission Hospital Road, Bilaspur

9630588100, 9893783828 coreacademybsp.in Core Academy