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1. If the angle between the vectors $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{4}$ and $|\vec{a} \times \vec{b}| = 1$, then $\vec{a} \cdot \vec{b}$ is equal to

(a) -1
(b) 1 ✓
(c) $\frac{1}{\sqrt{2}}$
(d) $\sqrt{2}$

$ab\cos\frac{\pi}{4} = ?$
$ab\sin\frac{\pi}{4} = 1 \Rightarrow ab\cos\frac{\pi}{4} = 1$
Ans = B (1)

2. $\vec{a}$ and $\vec{b}$ are two non-zero vectors such that the projection of $\vec{a}$ on $\vec{b}$ is 0. The angle between $\vec{a}$ and $\vec{b}$ is :

(a) $\frac{\pi}{2}$ ✓
(b) π
(c) $\frac{\pi}{4}$
(d) 0

$\vec{a} \cdot \hat{b} = 0$
$\Rightarrow a \times 1 \times \cos\theta = 0$
$\Rightarrow \theta = \frac{\pi}{2}$
A

3. In ΔABC, $\vec{AB} = \hat{i} + \hat{j} + 2\hat{k}$ and $\vec{AC} = 3\hat{i} - \hat{j} + 4\hat{k}$. If D is mid-point of BC, then vector $\vec{AD}$ is equal to :

(a) $4\hat{i} + 6\hat{k}$
(b) $2\hat{i} - 2\hat{j} + 2\hat{k}$
(c) $\hat{i} - \hat{j} + \hat{k}$
(d) $2\hat{i} + 3\hat{k}$ ✓

$\vec{AD} = \frac{1}{2}(\vec{AB} + \vec{AC})$
$= \frac{1}{2}(4\hat{i} + 0\hat{j} + 6\hat{k})$
(D) $= 2\hat{i} + 0\hat{j} + 3\hat{k}$

4. The equation of a line passing through point $(2, -1, 0)$ and parallel to the line $\frac{x}{1} = \frac{y-1}{2} = \frac{2-z}{2}$ is :

(a) $\frac{x+2}{1} = \frac{y-1}{2} = \frac{z}{2}$
(b) $\frac{x-2}{1} = \frac{y-1}{2} = \frac{z}{2}$
(c) $\frac{x+2}{1} = \frac{y-1}{2} = \frac{z}{-2}$
(d) $\frac{x-2}{1} = \frac{y+1}{2} = \frac{z}{-2}$ ✓

Point = 2, −1, 0
drs = 1, 2, −2
eq $\frac{x-2}{1} = \frac{y+1}{2} = \frac{z}{-2}$
(D)

5. X and Y are independent events such that $P(X \cap \bar{Y}) = \frac{2}{5}$ and $P(X) = \frac{3}{5}$. Then P(Y) is equal to :

(a) $\frac{2}{3}$
(b) $\frac{2}{5}$
(c) $\frac{1}{3}$ ✓
(d) $\frac{1}{5}$

Sol
$P(X \cap \bar{Y}) = \frac{2}{5}$
$\Rightarrow P(X)(1 - P(Y)) = \frac{2}{5}$
$\Rightarrow \frac{3}{5}(1 - P(Y)) = \frac{2}{5}$
$P(Y) = \frac{1}{3} \Rightarrow C$

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6. The value of k for which function $f(x)=\begin{cases} x^2, & x \geq 0 \\ kx, & x < 0 \end{cases}$ is differentiable at $x = 0$ is :

(a) 1 (b) 2

(c) any real number (d) 0

$f'(0^-) = f'(0^+)$
$K = 2(0^-)$
$K = 0 \Rightarrow D$

7. If $y = \dfrac{\cos x - \sin x}{\cos x + \sin x}$, then $\dfrac{dy}{dx}$ is :

(a) $-\sec^2\left(\dfrac{\pi}{4} - x\right)$ (b) $\sec^2\left(\dfrac{\pi}{4} - x\right)$

(c) $\log\left|\sec\left(\dfrac{\pi}{4} - x\right)\right|$ (d) $-\log\left|\sec\left(\dfrac{\pi}{4} - x\right)\right|$

$y = \tan\left(\frac{\pi}{4} - x\right)$
$\frac{dy}{dx} = -\sec^2\left(\frac{\pi}{4} - x\right)$
(A)

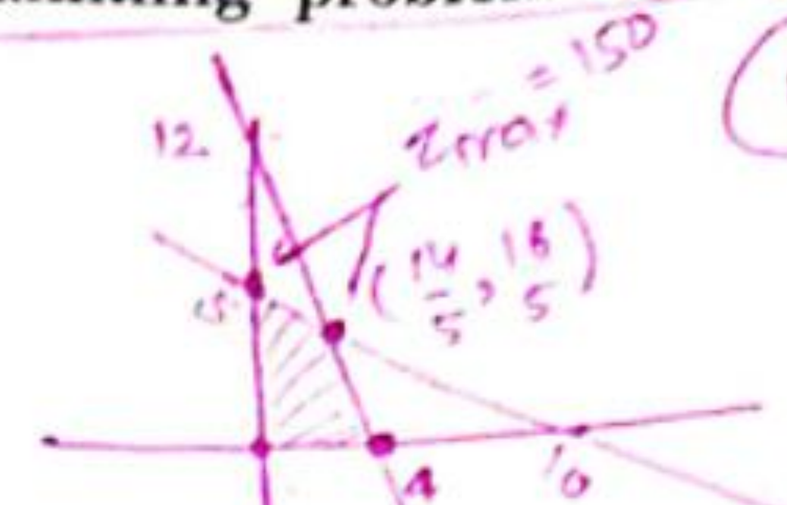
8. The number of feasible solutions of the linear programming problem given as

Maximize $z = 15x + 30y$ subject to constraints :
$3x + y \leq 12,\ x + 2y \leq 10,\ x \geq 0, y \geq 0$ is

(a) 1 (b) 2

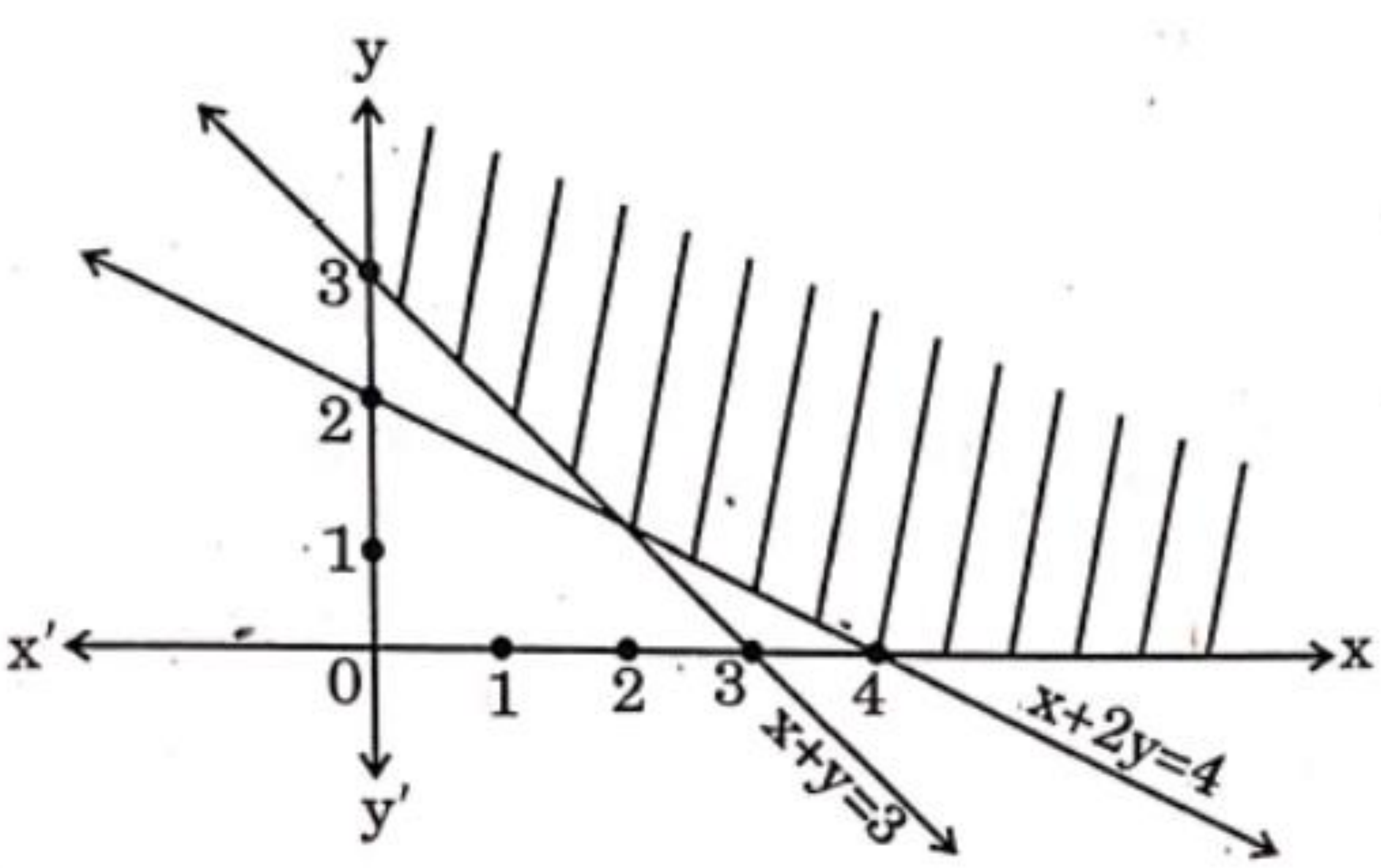
(c) 3 (d) infinite

(B)



at $(0,5)$ & $(\frac{14}{5}, \frac{18}{5})$ $z_{max} = 150$

9. The feasible region of a linear programming problem is shown in the figure below :



$x + 2y \geq 4$
$x + y \geq 3$
$x, y \geq 0$

(C)

Which of the following are the possible constraints ?

(a) $x + 2y \geq 4,\ x + y \leq 3,\ x \geq 0,\ y \geq 0$

(b) $x + 2y \leq 4,\ x + y \leq 3,\ x \geq 0,\ y \geq 0$

(c) $x + 2y \geq 4,\ x + y \geq 3,\ x \geq 0,\ y \geq 0$

(d) $x + 2y \geq 4,\ x + y \geq 3,\ x \leq 0,\ y \leq 0$

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10. A and B are square matrices of same order. If $(A+B)^2 = A^2 + B^2$, then :

(a) AB = BA
(b) AB = – BA
(c) AB = O
(d) BA = O

$(A+B)^2 = (A+B)(A+B)$
$\Rightarrow A^2 + B^2 = A^2 + AB + BA + B^2$
$\Rightarrow AB + BA = 0 \Rightarrow$ B

11. If A . (adj A) = $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, then the value of $|A| + |\text{adj } A|$ is equal to :

(a) 12
(b) 9
(c) 3
(d) 27

$A \cdot \text{Adj} A = |A| \cdot I$
$\Rightarrow |A| = 3, \; |\text{Adj} A| = 3^2 = |A|^2$
So $|A| + |\text{Adj} A| = 3 + 9 = 12$

(A)

12. A and B are skew-symmetric matrices of same order. AB is symmetric, if :

(a) AB = O
(b) AB = – BA
(c) AB = BA
(d) BA = O

Given $A^T = -A$, $B^T = -B$
we want $(AB)^T = AB$
So $(AB)^T = B^T \cdot A^T$
$= BA$
So $BA = AB$ is condition

(C)

13. For what value of $x \in \left[0, \frac{\pi}{2}\right]$, is $A + A' = \sqrt{3}\, I$, where

$A = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$?

(a) $\frac{\pi}{3}$
(b) $\frac{\pi}{6}$
(c) 0
(d) $\frac{\pi}{2}$

$A + A' = \begin{pmatrix} 2\cos x & 0 \\ 0 & 2\cos x \end{pmatrix} = \sqrt{3}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$\Rightarrow \cos x = \frac{\sqrt{3}}{2}$

$\Rightarrow x = \frac{\pi}{6} \Rightarrow$ B

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14. Let A be the area of a triangle having vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3). Which of the following is correct ?

(a) $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \pm A$

(b) ✓ $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \pm 2A$

(c) $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \pm \frac{A}{2}$

(d) $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}^2 = A^2$

formula

$A = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

15. $\int 2^{x+2}dx$ is equal to :

(a) $2^{x+2} + C$

(b) $2^{x+2}\log 2 + C$

(c) ✓ $\frac{2^{x+2}}{\log 2} + C$

(d) $2 \cdot \frac{2^x}{\log 2} + C$

formula

16. $\int e^{-x}\left(\frac{x+1}{x^2}\right)dx$ is equal to :

(a) $\frac{e^{-x}}{x} + C$

(b) $\frac{e^{x}}{x} + C$

(c) $\frac{e^{x}}{x^2} + C$

(d) ✓ $-\frac{e^{-x}}{x} + C$

$I = \int e^{-x}\left(\frac{1}{x} + \frac{1}{x^2}\right)dx$

$-x = t \Rightarrow dx = -dt$

$I = \int e^{t}\left(-\frac{1}{t} + \frac{1}{t^2}\right)(-dt)$

$= -e^{t}\left(-\frac{1}{t}\right) + c$

(D) $= -\frac{e^{-x}}{x} + c$

17. The value of $\int_0^{\pi/2} \log \tan x \, dx$ is :

(a) $\frac{\pi}{2}$

(b) ✓ 0

(c) $-\frac{\pi}{2}$

(d) 1

$I = \int_0^{\pi/2} \log \tan x \, dx$

$I = \int_0^{\pi/2} \ln(\cot x)\, dx$

$2I = \int_0^{\pi/2} \ln 1 = 0$

$I = 0$ B

18. What is the product of the order and degree of the differential equation

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18. What is the product of the order and degree of the differential equation $\frac{d^2y}{dx^2}\sin y + \left(\frac{dy}{dx}\right)^3 \cos y = \sqrt{y}$?

order = 2
degree = 1

(a) 3 (b) 2 ✓
(c) 6 (d) not defined

19. *Assertion (A) :* Range of $[\sin^{-1}x + 2\cos^{-1}x]$ is $[0, \pi]$. F (as range = $[\frac{\pi}{2}, \frac{3\pi}{2}]$)

Reason (R) : Principal value branch of $\sin^{-1}x$ has range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. T

(D)

20. *Assertion (A) :* A line through the points (4, 7, 8) and (2, 3, 4) is parallel to a line through the points (– 1, – 2, 1) and (1, 2, 5). T (dr's are same)

Reason (R): Lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$ are parallel if $\vec{b_1} \cdot \vec{b_2} = 0$. F

($\vec{b_1} = \lambda\vec{b_2}$ is T)

(C)

This section comprises very short answer (VSA) type questions of 2 marks each.

21. (a) If $y = x^{\frac{1}{x}}$, then find $\frac{dy}{dx}$ at $x = 1$.

Ans = 1

$\ln y = \frac{1}{x}\ln x$

$\frac{dy}{dx} = x^{\frac{1}{x}}\left(\frac{1-\ln x}{x^2}\right) \Rightarrow \left.\frac{dy}{dx}\right)_{x=1} = 1$

OR

(b) If $x = a\sin 2t$, $y = a(\cos 2t + \log\tan t)$, then find $\frac{dy}{dx}$.

Ans = cot 2t

$\frac{dx}{dt} = 2a\cos 2t$, $\frac{dy}{dt} = a\left(-2\sin 2t + \frac{\sec^2 t}{\tan t}\right) \Rightarrow$

$\frac{dy}{dx} = \frac{a\left(-2\sin 2t + \frac{1}{\sin t\cos t}\right)}{2a\cos 2t}$

$= \frac{2 - 2\sin^2 2t}{2\cos 2t\sin 2t} = \cot 2t$

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22. If $\vec{r} = 3\hat{i} - 2\hat{j} + 6\hat{k}$, find the value of $(\vec{r} \times \hat{j}) \cdot (\vec{r} \times \hat{k}) - 12$.

Ans = zero

23. Find the direction cosines of the line whose Cartesian equations are $5x - 3 = 15y + 7 = 3 - 10z$.

Eqⁿ: $\frac{x - 3/5}{\frac{1}{5}} = \frac{y + 7/15}{1/15} = \frac{z - 3/10}{-1/10} \Rightarrow$ drs $= 6, 2, -3$
dcs $= 6/7, 2/7, \frac{-3}{7}$

24. Find the points on the curve $6y = x^3 + 2$ at which ordinate is changing 8 times as fast as abscissa.

$\frac{dy}{dx} = 8 \Rightarrow \frac{3x^2}{6} = 8 \Rightarrow x^2 = 16 \Rightarrow x = \pm 4$
$\Rightarrow$ Points $(4, 11)$, $\left(-4, \frac{-31}{3}\right)$

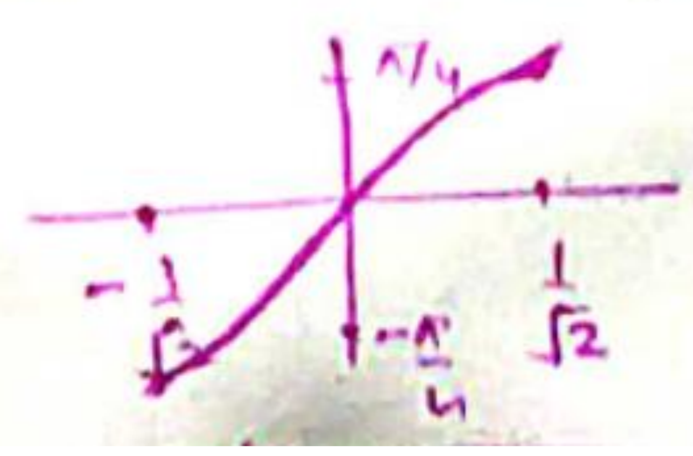
25. (a) Evaluate : $3\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) + 2\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) + \cos^{-1}(0)$

$= 3\frac{\pi}{4} + 2 \times \frac{\pi}{6} + \frac{\pi}{2} = \frac{19\pi}{12}$ Ans.

OR

(b) Draw the graph of $f(x) = \sin^{-1} x$, $x \in \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$. Also, write range of f(x).

Graph:



, range $= \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

26. (a) A pair of dice is thrown simultaneously. If X denotes the absolute difference of numbers obtained on the pair of dice, then find the probability distribution of X.

Ans.

X	0	1	2	3	4	5
P(X)	$\frac{6}{36}$	$\frac{10}{36}$	$\frac{8}{36}$	$\frac{6}{36}$	$\frac{4}{36}$	$\frac{2}{36}$

OR

(b) There are two coins. One of them is a biased coin such that P (head) : P (tail) is 1 : 3 and the other coin is a fair coin. A coin is selected at random and tossed once. If the coin showed head, then find the probability that it is a biased coin.

Biased Coin $P(H)=\frac{1}{4}$ $P(T)=\frac{3}{4}$, $P\left(\frac{\text{Coin is biased}}{\text{It shows Head}}\right) = \frac{P(B)P(H)}{P(B)P(H)+P(N)P(H)}$

$= \frac{\frac{1}{2}\times\frac{1}{4}}{\frac{1}{2}\times\frac{1}{4}+\frac{1}{2}\times\frac{1}{2}} = \frac{1}{3}$ Ans.

27. (a) Find the general solution of the differential equation :

$$\frac{d}{dx}(xy^2) = 2y(1+x^2) \Rightarrow 2xy\frac{dy}{dx}+y^2 = 2y+2x^2y$$

$\Rightarrow \frac{dy}{dx}+\frac{y}{2x} = \frac{1}{x}+x$

$\Rightarrow IF = e^{\int\frac{1}{2x}dx} = \sqrt{x}$, sol^n $y(\sqrt{x}) = \int\sqrt{x}\left(x+\frac{1}{x}\right)dx$

$\Rightarrow y\sqrt{x} = 2\sqrt{x}+\frac{2}{5}x^2\sqrt{x}+C$

$\Rightarrow y = 2+\frac{2x^2}{5}+\frac{C}{\sqrt{x}}$ Ans.

OR

(b) Solve the following differential equation :

$$xe^{\frac{y}{x}} - y + x\frac{dy}{dx} = 0 \Rightarrow \frac{y}{x} = t$$

$\frac{dy}{dx} = x\frac{dt}{dx}+t$

$\Rightarrow e^t - y + y + x\frac{dt}{dx} = 0 \Rightarrow \frac{dt}{dx} = -\frac{e^t}{x} \Rightarrow \int e^{-t}dt = \int -\frac{dx}{x}$

$\Rightarrow e^{-t} = \ln x + C$

$\Rightarrow e^{-y/x} = \ln x + C$

28. Evaluate :

$$\int_0^{\pi/4} \log(1+\tan x)\,dx = \frac{\pi}{8}\ln 2$$

28 $I = \int_0^{\pi/4} \ln(1+\tan x)\,dx$ —①

$I = \int_0^{\pi/4} \ln\left(1+\tan\left(\frac{\pi}{4}-x\right)\right)dx$

$= \int_0^{\pi/4} \ln\left(1+\frac{1-\tan x}{1+\tan x}\right)dx = \int_0^{\pi/4} \ln\left(\frac{2}{1+\tan x}\right)dx$ —②

① + ② $\Rightarrow$

$2I = \int_0^{\pi/4} \ln 2\,dx$

$2I = \ln 2 \times \frac{\pi}{4}$

$I = \frac{\pi}{8}\ln 2$ Ans.

P.T.O.

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29. (a) Find :

$$\int \frac{\cos x}{\sin 3x} dx$$

OR

(b) Find :

$$\int x^2 \log (x^2 + 1)\, dx$$

New

1

29 (a) $\int \frac{\cos x}{\sin 3x} dx = \int \frac{\cos x\, dx}{3\sin x - 4\sin^3 x} = I$

$\sin x = t$

$\cos x\, dx = dt$

$\Rightarrow \int \frac{\cos x}{\sin 3x} dx = \int \frac{dt}{3t - 4t^3} = \int \frac{dt}{t(3-4t^2)} = \int \frac{1}{3}\left(\frac{1}{t} + \frac{4t}{3-4t^2}\right) dt$

$I = \int \frac{1}{3t} dt - \frac{4}{3}\int \frac{t\,dt}{3-4t^2} = \int \frac{1/3}{t} dt + \frac{1}{6}\int \frac{8t}{3-4t^2} dt$

$= \frac{1}{3}\ln t - \frac{1}{6}\ln(3-4t^2) + C$

$\int \frac{\cos x}{\sin 3x} dx = \boxed{\frac{1}{3}\ln \sin x - \frac{1}{6}\ln(3-4\sin^2 x) + C}$ Ans.

29(b) $I = \int x^2 \ln(1+x^2)\, dx$ (using by parts)

$I = \ln(1+x^2)\int x^2 dx - \int \frac{2x}{1+x^2}\cdot\frac{x^3}{3} dx$

$= \frac{x^3}{3}\ln(1+x^2) - \frac{2}{3}\int \frac{x^4}{1+x^2} dx$ manipulating

$= \frac{x^3}{3}\ln(1+x^2) + \frac{2}{3}\int \frac{-x^4+1-1}{1+x^2} dx$

$= \frac{x^3}{3}\ln(1+x^2) + \frac{2}{3}\int \frac{1-x^4}{1+x^2} dx - \frac{2}{3}\int \frac{1}{1+x^2} dx$

$= \frac{x^3}{3}\ln(1+x^2) + \frac{2}{3}\int (1-x^2) dx - \frac{2}{3}\tan^{-1} x$

$\boxed{I = \frac{x^3}{3}\ln(1+x^2) + \frac{2}{3}x - \frac{2}{9}x^3 - \frac{2}{3}\tan^{-1} x + C}$ Ans.

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30. Find :

$$\int_1^4 \frac{1}{\sqrt{2x+1}-\sqrt{2x-1}}\,dx = \frac{26+7\sqrt{7}-3\sqrt{3}}{6} \text{ Ans.}$$

(30) $I = \int_1^4 \frac{1}{\sqrt{2x+1}-\sqrt{2x-1}}\,dx$ — rationalisation

$$I = \int_1^4 \frac{\sqrt{2x+1}+\sqrt{2x-1}}{2}\,dx$$

$$\Rightarrow 2I = \left[\frac{2}{3}\frac{(2x+1)^{3/2}}{2} + \frac{2}{3}\frac{(2x-1)^{3/2}}{2}\right]_1^4$$

$$6I = \left[(2x+1)^{3/2} + (2x-1)^{3/2}\right]_1^4$$

$$6I = \left(9^{3/2} + 7^{3/2}\right) - \left(3^{3/2} + 1^{3/2}\right)$$

$$= 27 + 7\sqrt{7} - 3\sqrt{3} - 1$$

$$6I = 26 + 7\sqrt{7} - 3\sqrt{3} \Rightarrow \boxed{I = \frac{26+7\sqrt{7}-3\sqrt{3}}{6}} \text{ Ans.}$$

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31. Solve the following linear programming problem graphically :

Minimize $z = x + 2y$

subject to the constraints

$2x + y \geq 3,$

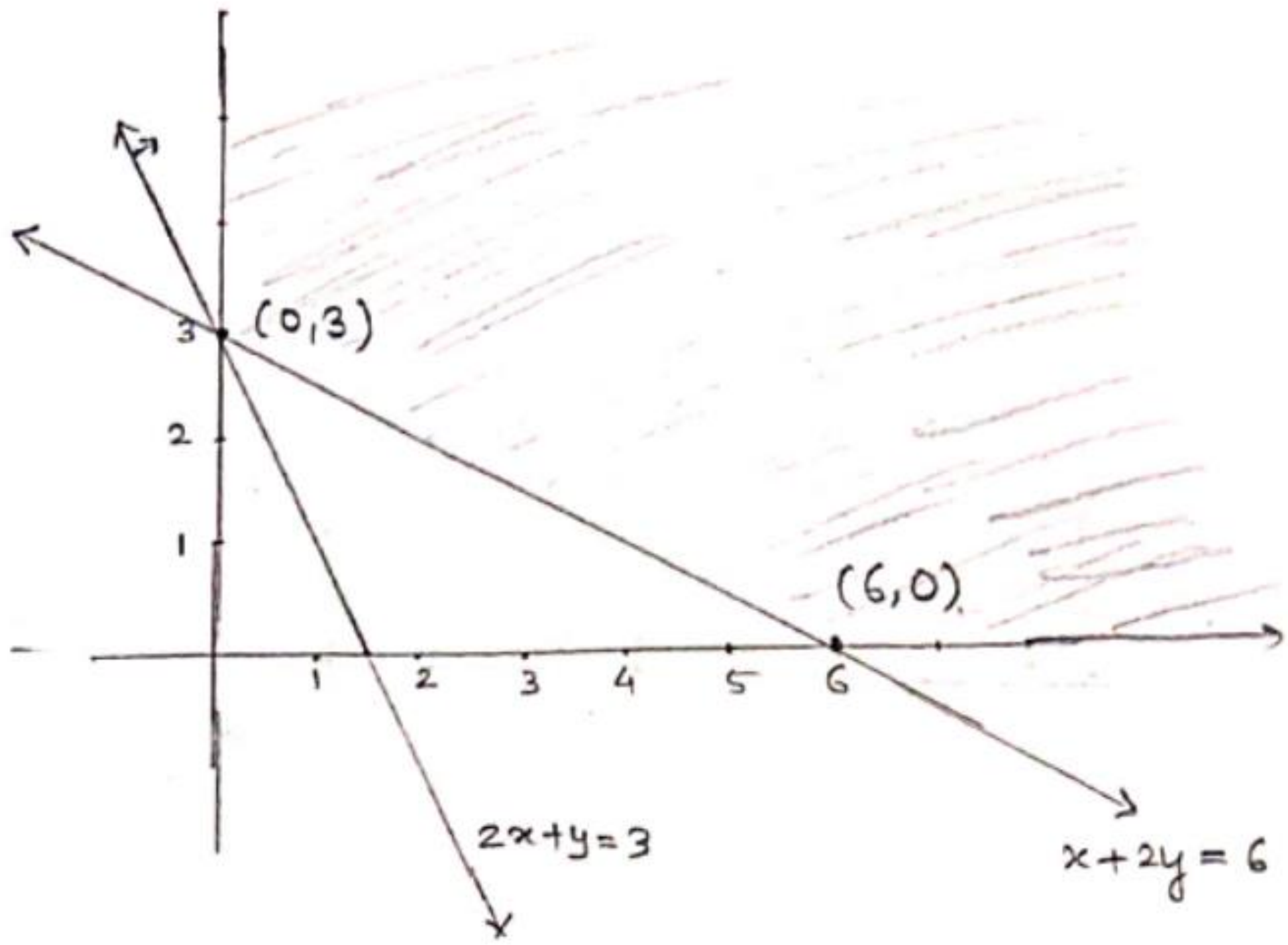
$x + 2y \geq 6,$

$x \geq 0,$

$y \geq 0.$

$z_{min} = 6$ at $(6, 0)$ & $(0, 3)$

(31) (3)



$z = x + 2y$ at $(0, 3)$ $z = 6$

at $(6, 0)$, $z = 6$

So $z_{min} = 6$ at $(0, 3)$ & $(6, 0)$.

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32. (a) Find the value of b so that the lines $\frac{x-1}{2}=\frac{y-b}{3}=\frac{z-3}{4}$ and $\frac{x-4}{5}=\frac{y-1}{2}=z$ are intersecting lines. Also, find the point of intersection of these given lines.

OR

(b) Find the equations of all the sides of the parallelogram ABCD whose vertices are A(4, 7, 8), B(2, 3, 4), C(– 1, – 2, 1) and D(1, 2, 5). Also, find the coordinates of the foot of the perpendicular from A to CD.

---

(32) (a) $L_1 : \frac{x-1}{2}=\frac{y-b}{3}=\frac{z-3}{4}=\lambda$ $\qquad$ $L_2 : \frac{x-4}{5}=\frac{y-1}{2}=\frac{z-0}{1}=\mu$

General point $(2\lambda+1,\ 3\lambda+b,\ 4\lambda+3)$ $\qquad$ Gen pt : $(5\mu+4,\ 2\mu+1,\ \mu)$

Lines are insecting $\Rightarrow (2\lambda+1, 3\lambda+b, 4\lambda+3) = (5\mu+4, 2\mu+1, \mu)$

$\Rightarrow 2\lambda+1=5\mu+4 \Rightarrow 2\lambda-5\mu=3$ — (1)

$3\lambda+b=2\mu+1 \Rightarrow 3\lambda-2\mu=1-b$ — (2)

$4\lambda+3=\mu \Rightarrow 4\lambda-\mu=-3$ — (3)

Solving (1) & (3) $\Rightarrow$

$2\lambda-5\mu=3$

$-\ 20\lambda-5\mu=-15$ $\quad$ (− + +)

$-18\lambda = 18 \Rightarrow$ $\boxed{\lambda=-1,\ \mu=-1}$

from (2) $\Rightarrow -3+2=1-b \Rightarrow$ $\boxed{b=2}$ Ans.

Point of intersection $(-1,-1,-1)=(-1,-1,-1)$

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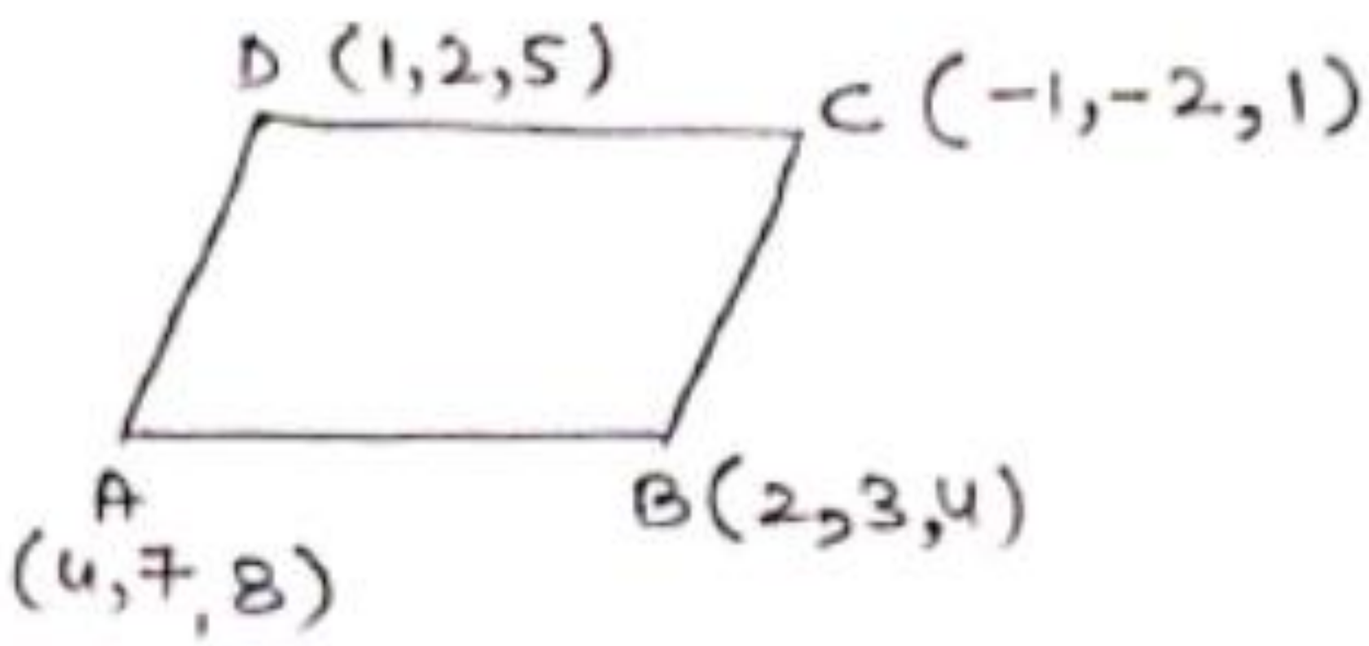
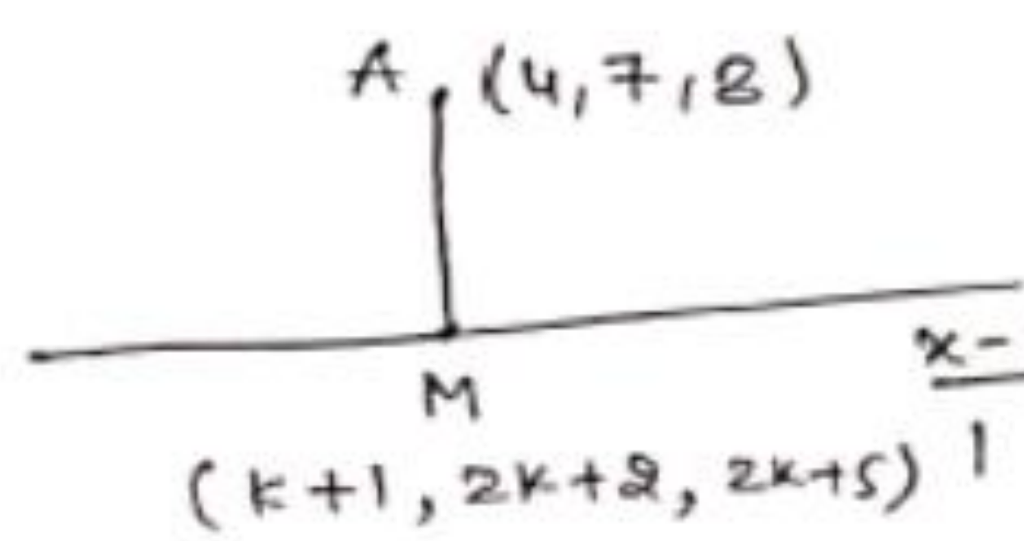
32.(b) (4)



	drs	Point	Eqn	
Eqn of AB :-	2,4,4 ≃ 1,2,2	(2,3,4)	$\frac{x-2}{1}=\frac{y-3}{2}=\frac{z-4}{2}$	Ans.
BC	3,5,3	(2,3,4)	$\frac{x-2}{3}=\frac{y-3}{5}=\frac{z-4}{3}$	Ans.
Eqn of CD	2,4,4 ≃ 1,2,2	(1,2,5)	$\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-5}{2}$	Ans.
Eqn of AD	3,5,3	(1,2,5)	$\frac{x-1}{3}=\frac{y-2}{5}=\frac{z-5}{3}$	Ans.

Foot of ⊥ from A to CD = M



$\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-5}{2}=k$

$M = (k+1,\ 2k+2,\ 2k+5)$

drs of AM $= k-3,\ 2k-5,\ 2k-3$

$AM \perp CD \Rightarrow (k-3)+2(2k-5)+2(2k-3)=0$

$\Rightarrow 9k-19=0 \Rightarrow k=\frac{19}{9}$

So $M = \left(\frac{19}{9}+1,\ \frac{38}{9}+2,\ \frac{38}{9}+5\right) = \left(\frac{28}{9},\ \frac{56}{9},\ \frac{83}{9}\right)$ Ans.

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33. Check whether a function $f : \mathbb{R} \rightarrow \left[-\frac{1}{2}, \frac{1}{2}\right]$ defined as $f(x) = \frac{x}{1+x^2}$ is one-one and onto or not. not one-one, f is onto

(33) $f : R \rightarrow \left[-\frac{1}{2}, \frac{1}{2}\right]$, $f(x) = \frac{x}{1+x^2}$

Check for one-one

Let for $x_1, x_2 \in R$

$f(x_1) = f(x_2)$

$\frac{x_1}{1+x_1^2} = \frac{x_2}{1+x_2^2}$

$\Rightarrow x_1 + x_1 x_2^2 = x_2 + x_1^2 x_2$

$\Rightarrow (x_1 - x_2) - x_1 x_2 (x_1 - x_2) = 0$

$\Rightarrow (x_1 - x_2)(1 - x_1 x_2) = 0$

$\Rightarrow x_1 x_2 = 1 \quad (x_1 \neq x_2)$

So for $2, \frac{1}{2}$ in R, $f(2) = f\left(\frac{1}{2}\right) = \frac{2}{5}$

$\Rightarrow$ **f is not one-one**

Ans.

Check for onto

Let $y = f(x) = \frac{x}{1+x^2}$

$\Rightarrow y = \frac{1}{x + \frac{1}{x}}$

$\forall x \in R_0$, $x + \frac{1}{x} \geq 2$ or ≤ -2

So $\frac{1}{x + \frac{1}{x}} \leq \frac{1}{2}$ or $\frac{1}{x + \frac{1}{x}} \geq -\frac{1}{2}$

and for $x = 0$, $\frac{x}{1+x^2} = 0$

So $\forall x \in R$, $\frac{x}{1+x^2} \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

So f(x) is onto, range = Codomain

Ans.

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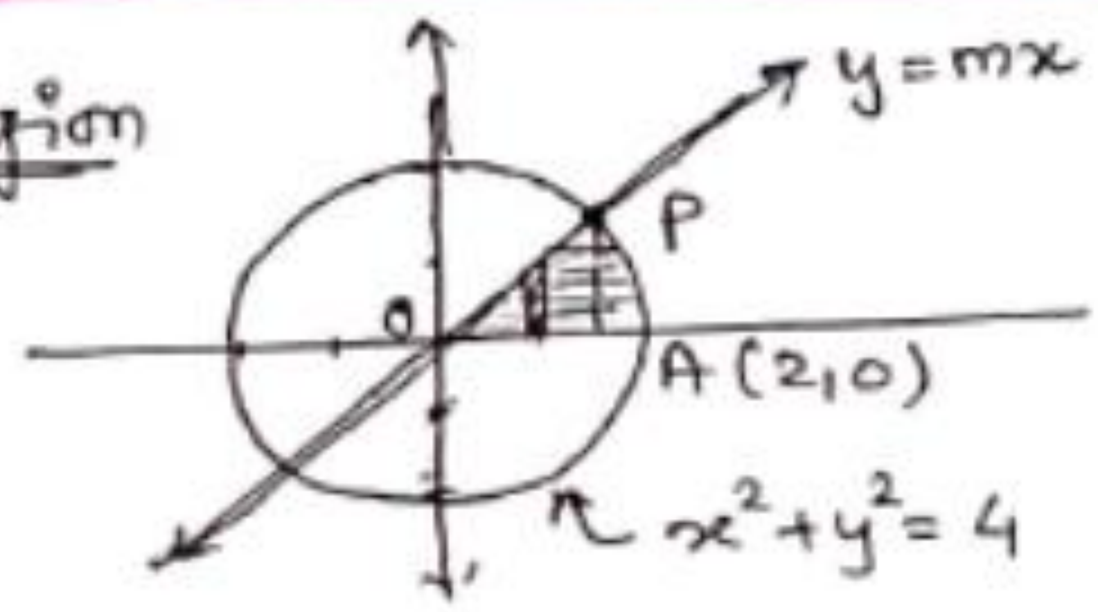
34. The area of the region bounded by the line $y = mx$ $(m > 0)$, the curve $x^2 + y^2 = 4$ and the x-axis in the first quadrant is $\frac{\pi}{2}$ units. Using integration, find the value of m. $\boxed{m=1}$ Ans.

(34) 1. Region



2. Point of intersection : P

$$x^2 + m^2x^2 = 4$$

$$x = \frac{2}{\sqrt{1+m^2}} = P$$

3. Area of Shaded reg = $\frac{\pi}{2}$

$$\int_0^{2/\sqrt{1+m^2}} y\,dx + \int_{\frac{2}{\sqrt{1+m^2}}}^{2} y\,dx = \pi/2$$

$$\Rightarrow \int_0^P mx\,dx + \int_P^2 \sqrt{4-x^2}\,dx = \pi/2$$

$$\Rightarrow \frac{m}{2}\left[x^2\right]_0^{\frac{2}{\sqrt{1+m^2}}} + \left[\frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_{\frac{2}{\sqrt{1+m^2}}}^{2} = \pi/2$$

$$\frac{m}{2}\left[\frac{4}{1+m^2} - 0\right] + \left[\frac{4}{2}\sin^{-1}(1)\right] - \left[\frac{1}{\sqrt{1+m^2}}\sqrt{4-\frac{4}{1+m^2}} + \frac{4}{2}\sin^{-1}\frac{1}{\sqrt{1+m^2}}\right] = \frac{\pi}{2}$$

$$\Rightarrow \frac{2m}{1+m^2} + \pi - \frac{2m}{1+m^2} - 2\sin^{-1}\frac{1}{\sqrt{1+m^2}} = \frac{\pi}{2}$$

$$\Rightarrow 2\sin^{-1}\frac{1}{\sqrt{1+m^2}} = \frac{\pi}{2} \Rightarrow \sin^{-1}\frac{1}{\sqrt{1+m^2}} = \frac{\pi}{4}$$

$$1 + m^2 = 2$$

$\boxed{m=1}$ Ans.

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35. (a) If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$, then show that $A^3 - 6A^2 + 7A + 2I = O$.

OR

(b) If $A = \begin{bmatrix} 3 & 2 \\ 5 & -7 \end{bmatrix}$, then find A^{-1} and use it to solve the following system of equations :

$3x + 5y = 11,\ 2x - 7y = -3.$

$A^{-1} = \frac{-1}{31}\begin{pmatrix} -7 & -2 \\ -5 & 3 \end{pmatrix}$, $x = 2,\ y = 1$

35 [a] $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}$ Show that $A^3 - 6A^2 + 7A + 2I = 0$

$$A^2 = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}\begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix} = \begin{pmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{pmatrix}$$

$$A^3 = \begin{pmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{pmatrix}\begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix} = \begin{pmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{pmatrix}$$

$$A^3 - 6A^2 + 7A + 2I = \begin{pmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{pmatrix} - \begin{pmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{pmatrix} + \begin{pmatrix} 7 & 0 & 14 \\ 0 & 14 & 7 \\ 14 & 0 & 21 \end{pmatrix} + \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$ Hence Proved

35 (b) $A = \begin{pmatrix} 3 & 2 \\ 5 & -7 \end{pmatrix}$

Finding A^{-1}

$C_A = \begin{pmatrix} -7 & -5 \\ -2 & 3 \end{pmatrix}$

$Adj(A) = \begin{pmatrix} -7 & -2 \\ -5 & 3 \end{pmatrix}$

$A^{-1} = \frac{1}{|A|} \cdot Adj(A)$ Ans.

$A^{-1} = \frac{1}{-31}\begin{pmatrix} -7 & -2 \\ -5 & 3 \end{pmatrix}$

Since $A^{-1} = \frac{-1}{31}\begin{pmatrix} -7 & -2 \\ -5 & 3 \end{pmatrix}$

$(A^T)^{-1} = \frac{-1}{31}\begin{pmatrix} -7 & -5 \\ -2 & 3 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} 3 & +5 \\ 2 & -7 \end{pmatrix}^{-1} = \frac{-1}{31}\begin{pmatrix} -7 & -5 \\ -2 & 3 \end{pmatrix}$

Solve $3x + 5y = 11$
$2x - 7y = -3$

$\Rightarrow \begin{pmatrix} 3 & 5 \\ 2 & -7 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 11 \\ -3 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 & 5 \\ 2 & -7 \end{pmatrix}^{-1}\begin{pmatrix} 11 \\ -3 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{-1}{31}\begin{pmatrix} -7 & -5 \\ -2 & 3 \end{pmatrix}\begin{pmatrix} 11 \\ -3 \end{pmatrix}$

$= \frac{-1}{31}\begin{pmatrix} -62 \\ -31 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$\Rightarrow$ $x = 2,\ y = 1$ Ans.

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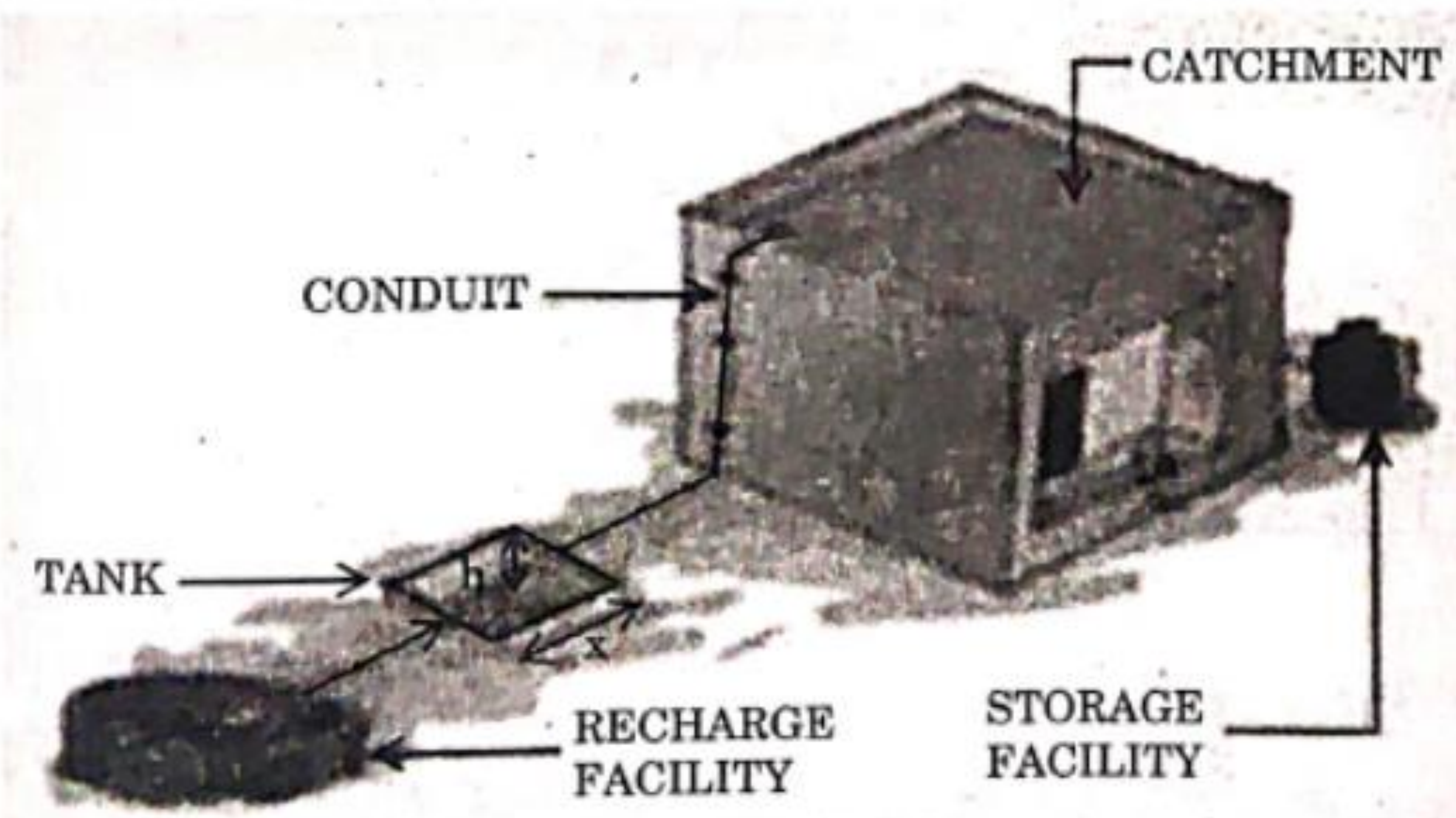
36. In order to set up a rain water harvesting system, a tank to collect rain water is to be dug. The tank should have a square base and a capacity of 250 m^3. The cost of land is ₹ 5,000 per square metre and cost of digging increases with depth and for the whole tank, it is ₹ 40,000 h^2, where h is the depth of the tank in metres. x is the side of the square base of the tank in metres.

ELEMENTS OF A TYPICAL RAIN WATER HARVESTING SYSTEM



Based on the above information, answer the following questions :

(i) Find the total cost C of digging the tank in terms of x. $C = 5000\left(x^2 + \frac{500,000}{x^4}\right)$ 1

(ii) Find $\frac{dC}{dx}$. $= 10^4\left(x - \frac{10^6}{x^3}\right)$ 1

(iii) (a) Find the value of x for which cost C is minimum. 2

$x = 10\sqrt{10}$ m

OR

(iii) (b) Check whether the cost function C(x) expressed in terms of x is increasing or not, where $x > 0$. 2

f is increasing for $x > 10\sqrt{10}$ and not for $x > 0$

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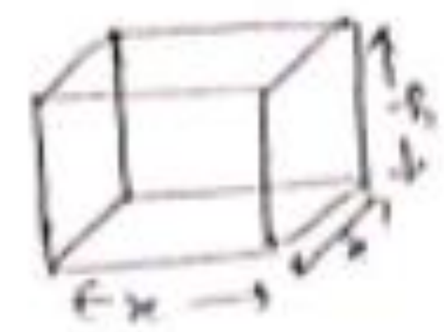
36 : Tank



Cost of land = Rs. 5000/m²
Cost of digging = 40,000 h^2 (h = height)

(i) $C = 5000x^2 + 40000h^2$

since $250 = x^2 h \Rightarrow h = \frac{250}{x^2}$

$C = 5000x^2 + 40000\cdot\left(\frac{250}{x^2}\right)^2$

$C = 5000x^2 + \frac{40{,}000 \times 62500}{x^4} = 5000x^2 + \frac{2500000000}{x^4}$

$= 5000\left(x^2 + \frac{500000}{x^4}\right)$ Ans.

(ii) ~~$\frac{dC}{dx} = 10000x - \ldots$~~

(ii) $\frac{dC}{dx} = 5000\left(2x - \frac{4 \times 500000}{x^3}\right)$ Ans.

(iii) a for min value of C, $\frac{dC}{dx} = 0$

$\Rightarrow x^4 = 1000000 = 10^6$

$x^2 = 10^3 = 1000$

since x = only one positive value of $\frac{dC}{dx}$ so C will be min for it, according to the question.

$x = \sqrt{1000} \Rightarrow x = 10\sqrt{10}$ m Ans.

(iii) b check C(x) is increasing for x > 0 or not

$\frac{dC}{dx} > 0$

$5000\left(2x - \frac{2000000}{x^3}\right) > 0$

$\Rightarrow 10^4\left(x - \frac{10^6}{x^3}\right) > 0$

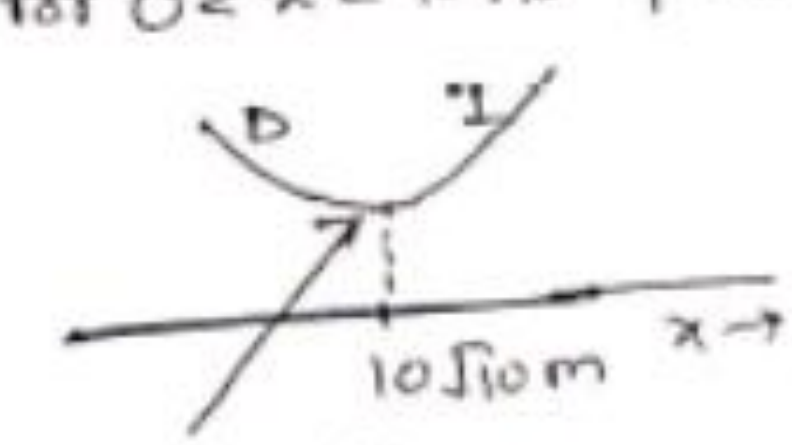
$\Rightarrow x^4 > 10^6 \quad (x>0)$

$\Rightarrow (x^2 - 10^3)(x^2 + 10^3) > 0$

$\Rightarrow x^2 > 10^3$

$\Rightarrow$ for $x > 10\sqrt{10}$ f is ↑

for $0 < x < 10\sqrt{10}$ f is ↓



Pt of minima

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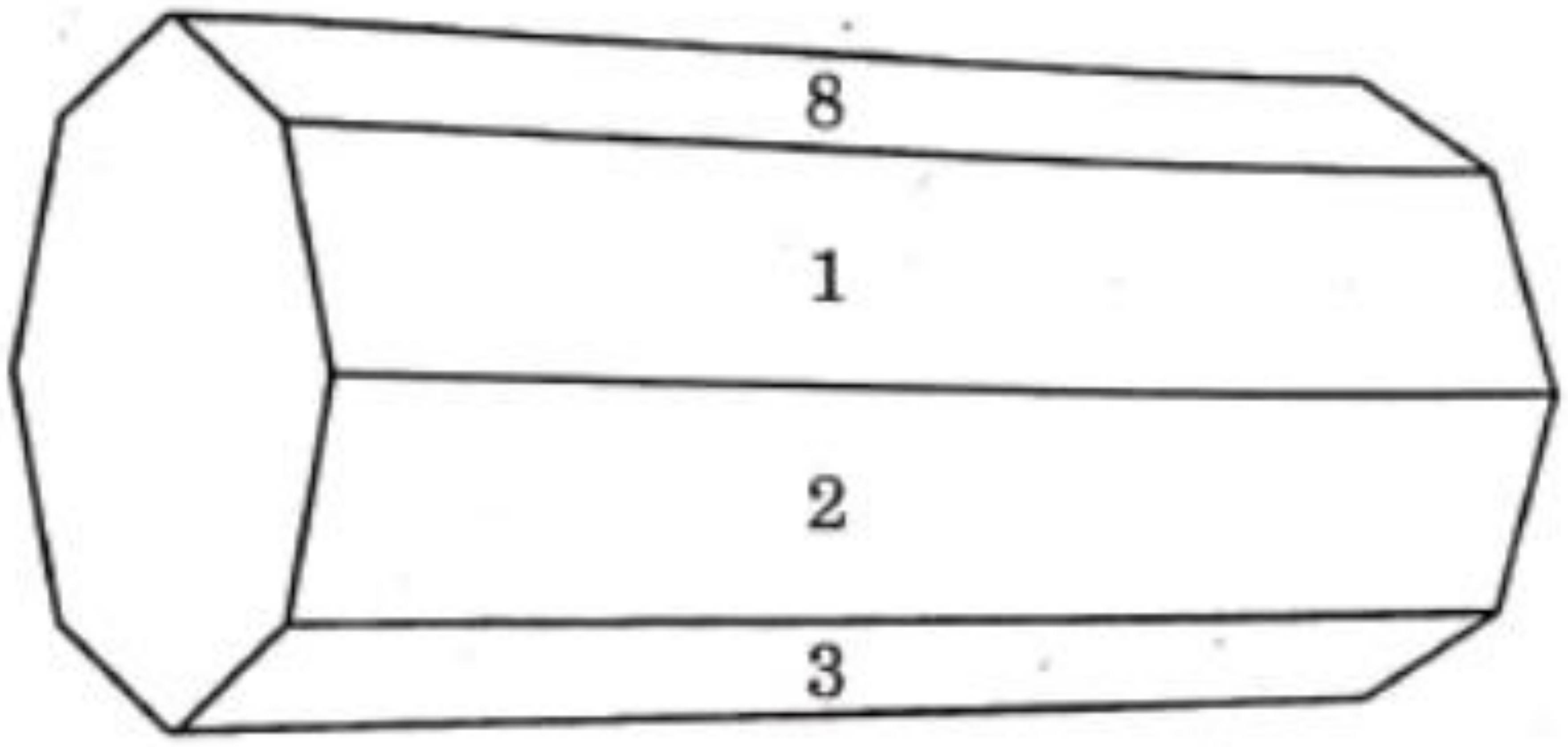
37. An octagonal prism is a three-dimensional polyhedron bounded by two octagonal bases and eight rectangular side faces. It has 24 edges and 16 vertices.



The prism is rolled along the rectangular faces and number on the bottom face (touching the ground) is noted. Let X denote the number obtained on the bottom face and the following table give the probability distribution of X.

X :	1	2	3	4	5	6	7	8
P(X) :	p	2p	2p	p	2p	p^2	$2p^2$	$7p^2 + p$

Based on the above information, answer the following questions :

(i) Find the value of p. $p = \frac{1}{10}$ 1

(ii) Find P(X > 6). $P(X > 6) = 0.19$ 1

(iii) (a) Find P(X = 3m), where m is a natural number. $= 0.21$ 2

OR

(iii) (b) Find the mean E(X). $= 4.06$ 2

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37 (i) $\Sigma p(x_i) = 1$

$\Rightarrow p + 2p + 2p + p + 2p + p^2 + 2p^2 + 7p^2 + p = 1$

$\Rightarrow 10p^2 + 9p - 1 = 0$

$10p^2 + 10p - p - 1 = 0$

$10p(p+1) - (p+1) = 0$

$p = -1$ or $p = \frac{1}{10}$ Ans.

(ii) $P(X > 6)$

$= P(X=7) + P(X=8)$

$= 2p^2 + 7p^2 + p$

$= 9p^2 + p$

$= \frac{9}{100} + \frac{1}{10}$

$= .09 + 0.1$

$P(X>6) = 0.19$ Ans.

(iii) a $P(X = 3m) = P(X=3) + P(X=6)$

$= 2p + p^2$

$= 2 \times \frac{1}{10} + \frac{1}{100}$

$= 0.2 + 0.01 = 0.21$ Ans.

OR

(iii) b Mean $E(X) = \Sigma x_i\, p(x_i)$

$= p + 2(2p) + 3(2p) + 4p + 10p + 6p^2 + 14p^2 + 56p^2 + 8p$

$= 33p + 76p^2$

$= \frac{33}{10} + \frac{76}{100}$

$= 3.3 + .76 = 4.06$ Ans.

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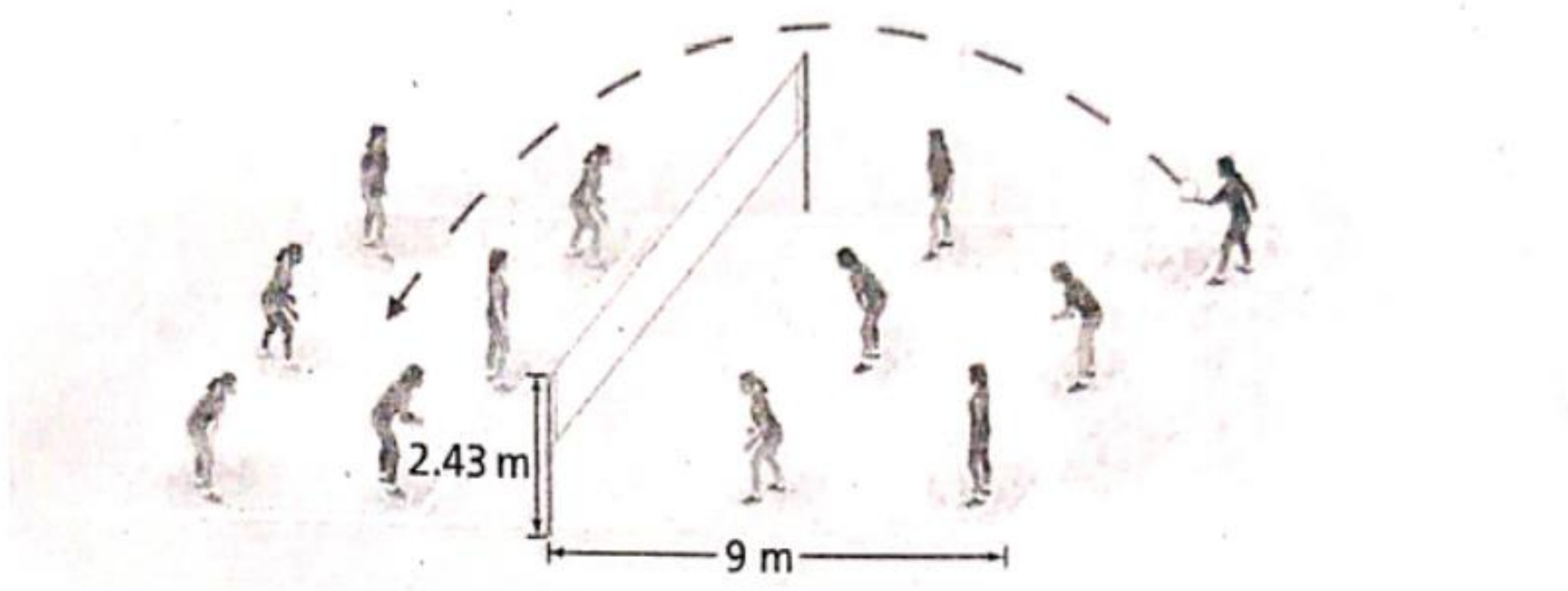
38. A volleyball player serves the ball which takes a parabolic path given by the equation $h(t) = -\frac{7}{2}t^2 + \frac{13}{2}t + 1$, where h(t) is the height of ball at any time t (in seconds), $(t \geq 0)$.



Based on the above information, answer the following questions :

(i) Is h(t) a continuous function ? Justify. Yes, it is poly. function **2**

(ii) Find the time at which the height of the ball is maximum. **2**

$t = \frac{13}{14}$ sec

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38 (i) Yes, it is polynomial function.

(ii) Time when height is maximum

$$h = -\frac{7}{2}t^2 + \frac{13}{2}t + 1$$

$$\frac{dh}{dt} = -7t + \frac{13}{2}, \quad \frac{d^2h}{dt^2} = -7 \quad (\text{maxima})$$

~~for increase~~

~~d^2h~~

$$\frac{dh}{dt} = 0 \Rightarrow t = \frac{13}{14} \text{ sec} \quad (\text{pt of maxima})$$

So $\boxed{\text{time} = \frac{13}{14} \text{ sec}}$ Ans.